Homework assignments

Self study	**Homework (to be handed in)**
Read through chapters 9 and 10 Example 10.2.2 Example 10.3.2	Exercise 9B: 1, 8 Exercise 10A: 3, 7 Exercise 10B: 5 Exercise 10C: 4, 8

10. Force as a vector quantity

- Understand the terms ‘resultant’ and ‘component’
- Be able to find the resultant of two or more forces
- Be able to find the components of a force in two given directions
- Appreciate that when a force is split into perpendicular components their magnitudes are equal to there resolved parts of the force in the given directions
- Be familiar with the notation of vector addition and with the representation of forces by column vectors
- Understand how the equilibrium of two or three forces can be expressed in vector notation
- Be able to represent the equilibrium of three forces with a triangle of forces.

10.1 Combining forces geometrically

What is a vector?

Displacement, velocity, acceleration, force, etc. are all **vector quantities.**

Need to describe them by means of a magnitude AND direction.

What about kinetic energy? **Scalar quantity.**

In print vectors are often written in bold, e.g. **P**

$\vec{P} \equiv \underline{P} \equiv \underset{\sim}{\mathrm{P}}$

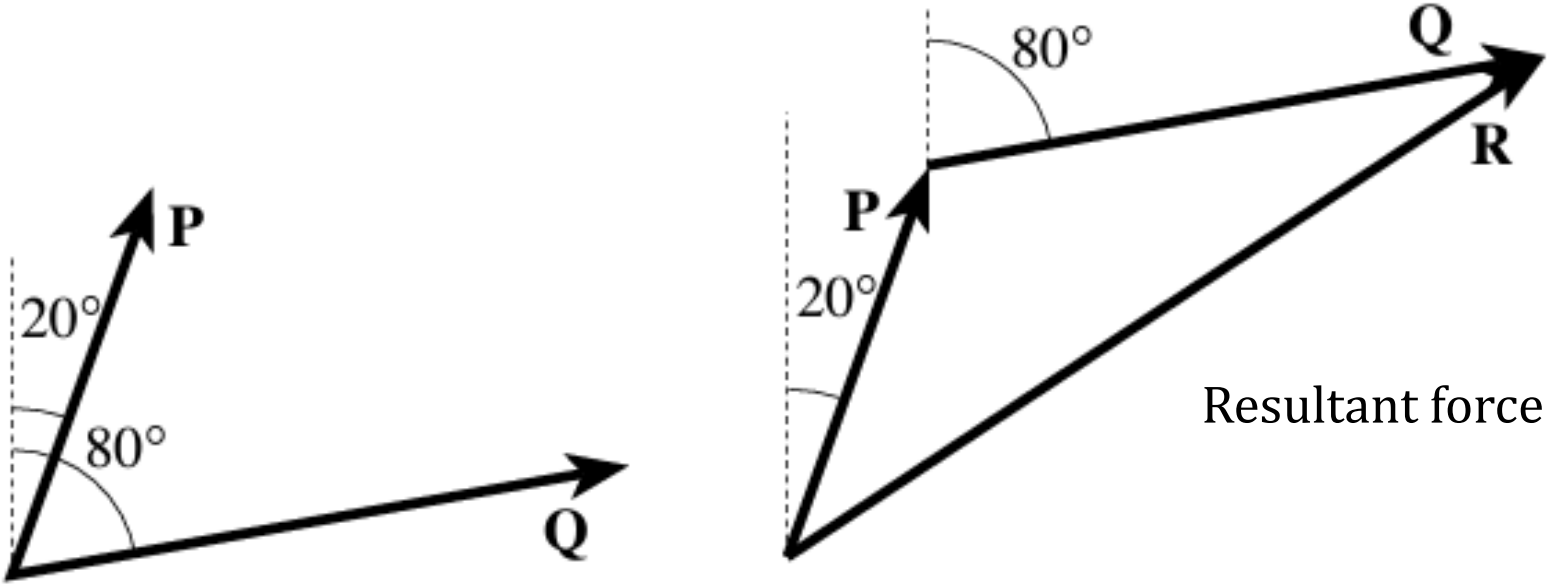
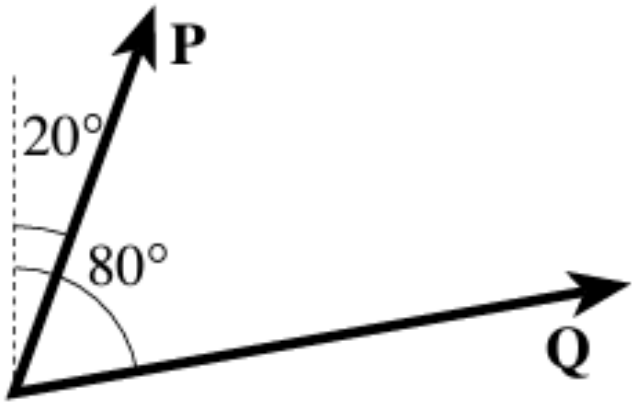
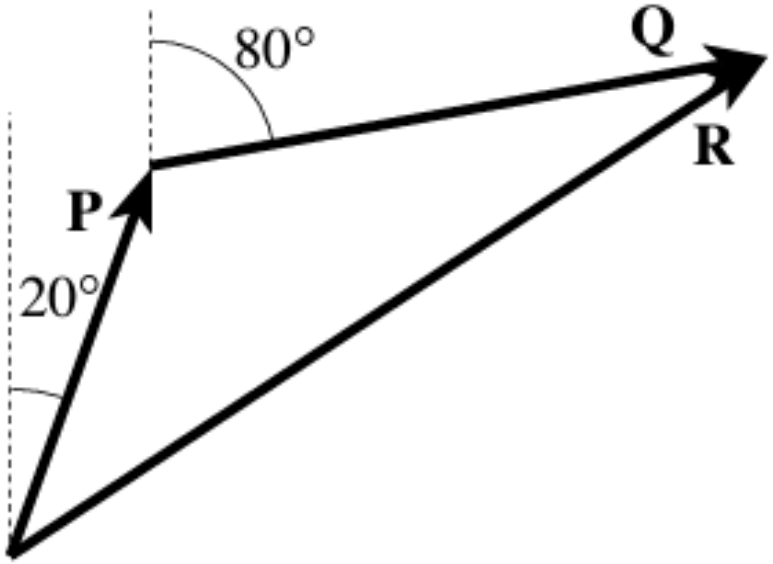


Fig. 10.2

Triangle law for combining forces If two forces **P** and **Q** are represented by arrows (on some scale) and the arrow representing **R** is obtained from these as in Fig. 10.2, then the single force **R** has exactly the same effect on a particle as the two forces **P** and **Q** acting together.



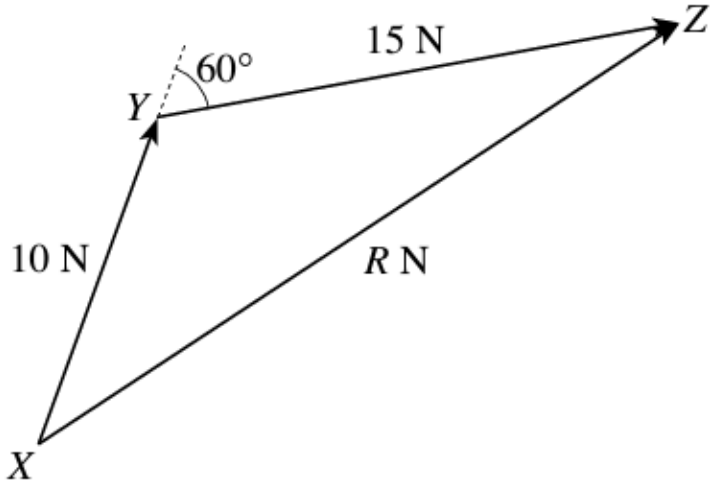


Example 10.1.1

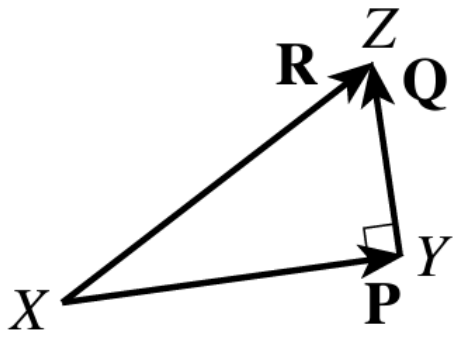
Find the resultant **R** of the two forces **P** and **Q** in Fig. 10.1 by calculation.

$P = 10$ N

Q= 15 N

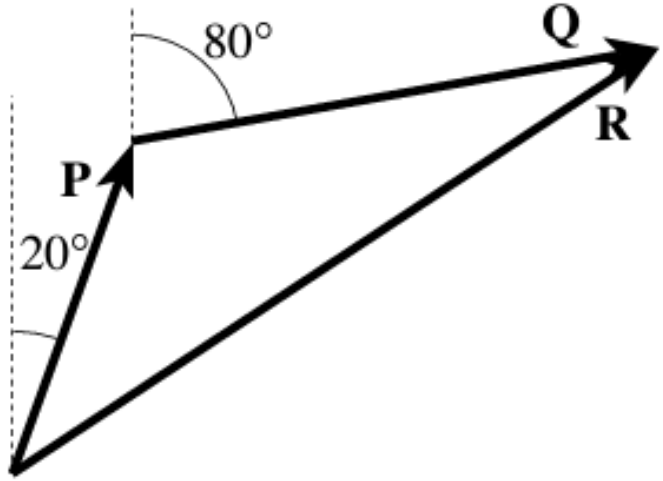


Special case if **P** and **Q** are perpendicular to each other



The resultant of two forces with magnitudes P and Q in perpendicular directions is a force of magnitude $\sqrt{P^2+Q^2}$ which makes an angle $\tan^{-1}\frac{Q}{P}$ with the direction of the force P.

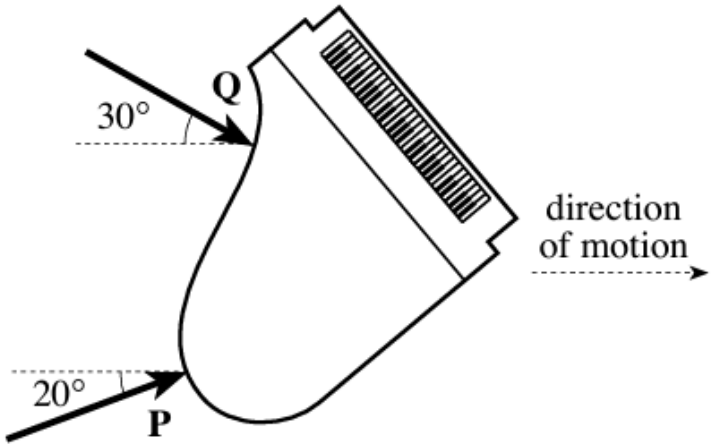
10.2 Splitting a force into components

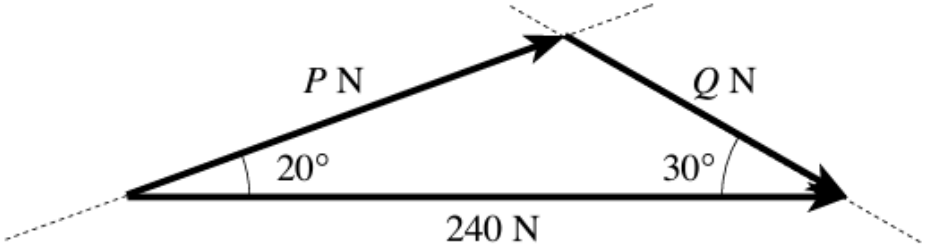


Let's say that you know **R** and want to calculate **Q** and **P**.

Example 10.2.1

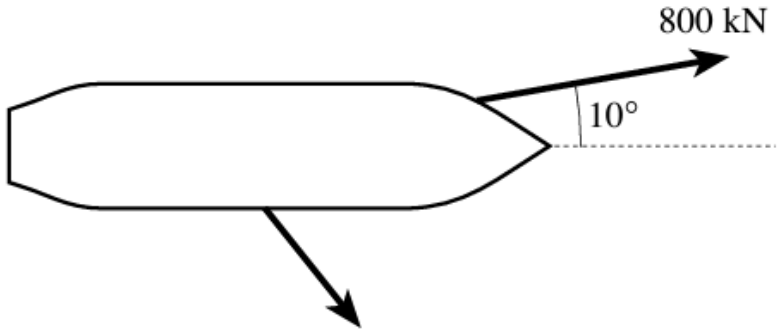
Two people are pushing a piano across a stage. This needs a force of $240\ \text{N}$. One person pushes at $20°$ to the left of the desired direction of motion, the other pushes at $30°$ to the right of it. How hard must each person push?

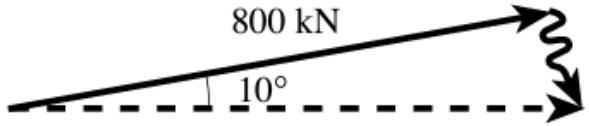




Example 10.2.2

A ship is towed along a narrow channel by cables attached to two tugboats. The more powerful tugboat produces a force of 800 kN ; its cable is at 10° to the direction of the channel. The other tugboat is to produce as small a force as possible. What should be the direction of the second tugboat's cable, and how large is the net forward force on the ship?



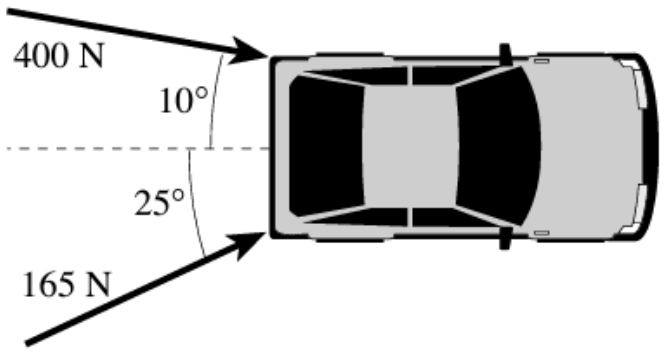


Self study

If a force is split into components in two perpendicular directions, the magnitude of each component is the resolved part of the force in that direction.

Class exercises
Exercise 10A pg. 149

2 A car is being pushed by two people. The magnitude and direction of the horizontal force exerted by each is shown in the diagram. Find the resultant of the two forces.

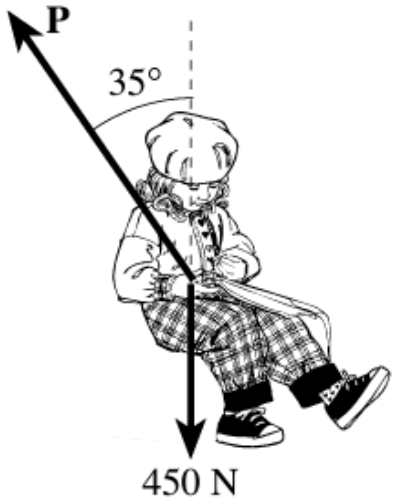


6 A child has weight 450 N. Whilst on a fairground ride the child is subject to a force **P** acting at 35° to the upward vertical, as shown in the diagram. Given that the resultant of **P** and the child's weight acts horizontally,

(a) find the magnitude of this resultant,

(b) state the direction of the child's acceleration,

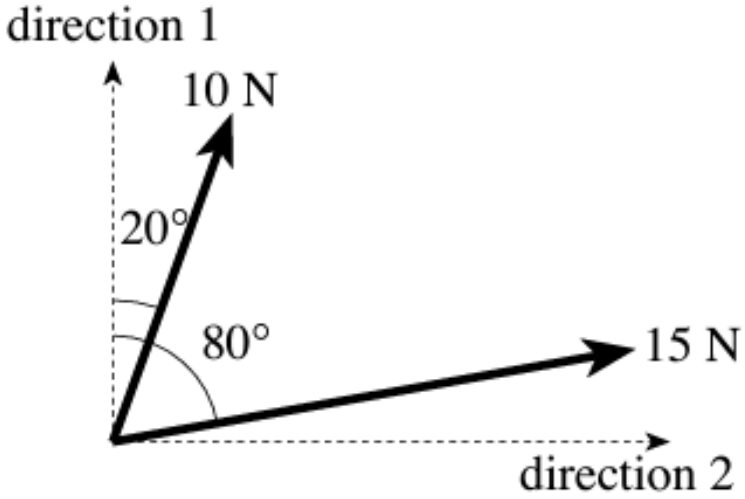
(c) find the magnitude of the child's acceleration.



10.3 Combining forces by perpendicular components

Example 10.3.1

Use this procedure to calculate the resultant of **P** and **Q** in Fig. 10.1



Step 1: Choose two directions at right angles (Choose your axes-system)

Step 2: Split each force into components in these directions

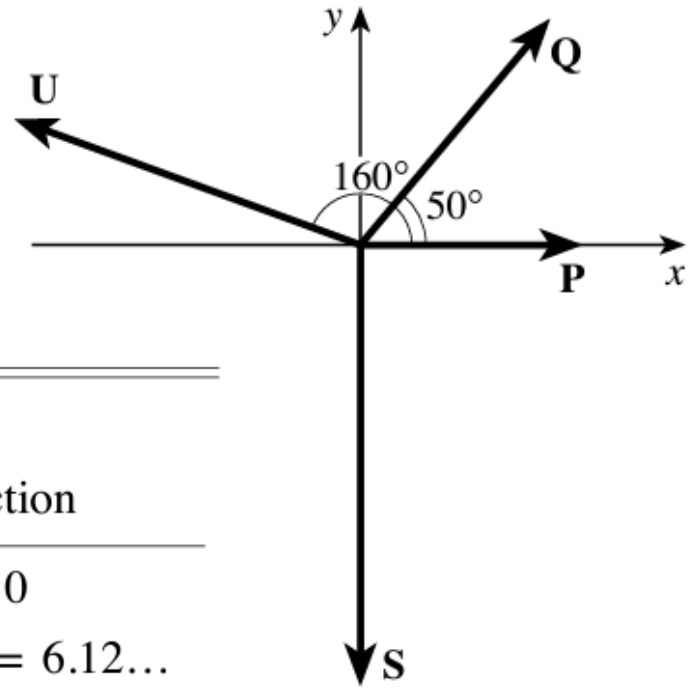
Step 3: For each direction, find the sum of the components you have calculated

Step 4: Find the resultant of the two sums

Example 10.3.2

The following four forces act on a particle (see Fig. 10.18). Find their resultant **V**.

$P = 6$ N
$Q = 8$ N
$S = 12$ N
$U = 10$ N



Force	Component in	
	x-direction	y-direction
P	6	0
Q	$8\cos 50° = 5.14\ldots$	$8\sin 50° = 6.12\ldots$
S	0	–12
U	$10\cos 160° = -9.39\ldots$	$10\sin 160° = 3.42\ldots$
V	1.74…	–2.45…

10.4 Using algebraic notation

Example 10.3.2

The following four forces act on a particle (see Fig. 10.18). Find their resultant $\mathbf{V}$.

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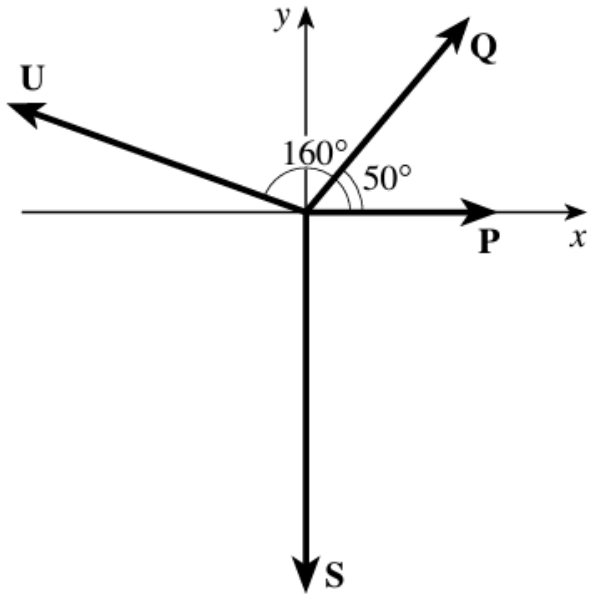

$\mathbf{V} = \mathbf{P} + \mathbf{Q} + \mathbf{S} + \mathbf{U} =$

10.4 Using algebraic notation

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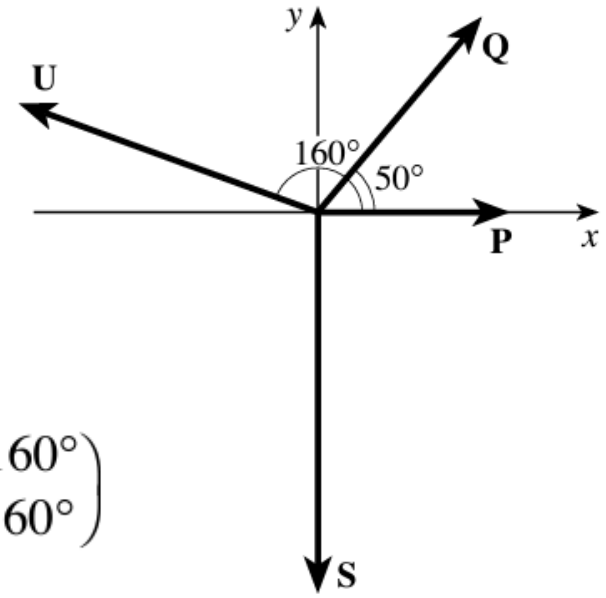

$$\mathbf{V} = \mathbf{P} + \mathbf{Q} + \mathbf{S} + \mathbf{U} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 8\cos 50° \\ 8\sin 50° \end{pmatrix} + \begin{pmatrix} 0 \\ -12 \end{pmatrix} + \begin{pmatrix} 10\cos 160° \\ 10\sin 160° \end{pmatrix}$$

10.4 Using algebraic notation

Example 10.3.2

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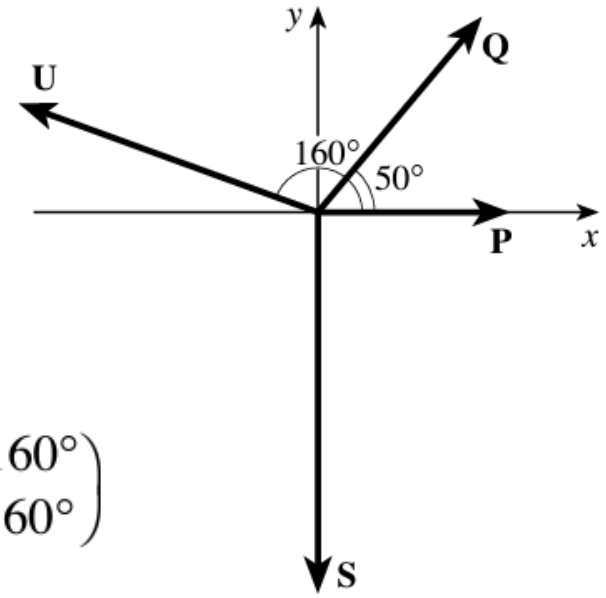

$$\mathbf{V} = \mathbf{P} + \mathbf{Q} + \mathbf{S} + \mathbf{U} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 8\cos 50° \\ 8\sin 50° \end{pmatrix} + \begin{pmatrix} 0 \\ -12 \end{pmatrix} + \begin{pmatrix} 10\cos 160° \\ 10\sin 160° \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 5.14\ldots \\ 6.12\ldots \end{pmatrix} + \begin{pmatrix} 0 \\ -12 \end{pmatrix} + \begin{pmatrix} -9.39\ldots \\ 3.42\ldots \end{pmatrix}$$

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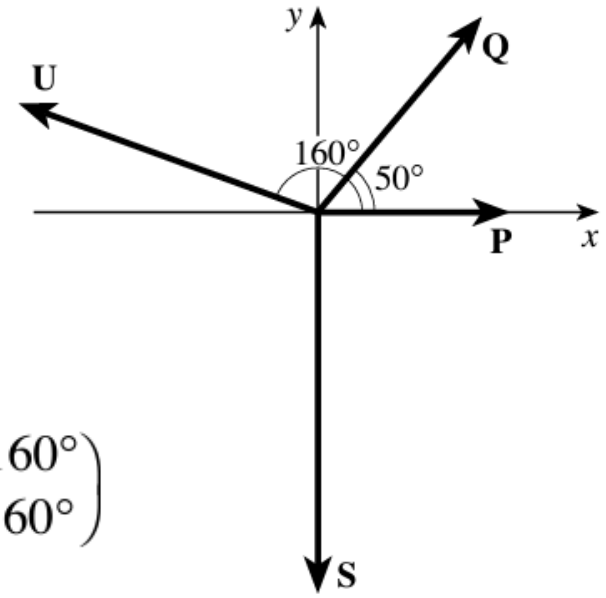


$$\mathbf{V} = \mathbf{P} + \mathbf{Q} + \mathbf{S} + \mathbf{U} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 8\cos 50° \\ 8\sin 50° \end{pmatrix} + \begin{pmatrix} 0 \\ -12 \end{pmatrix} + \begin{pmatrix} 10\cos 160° \\ 10\sin 160° \end{pmatrix}$$

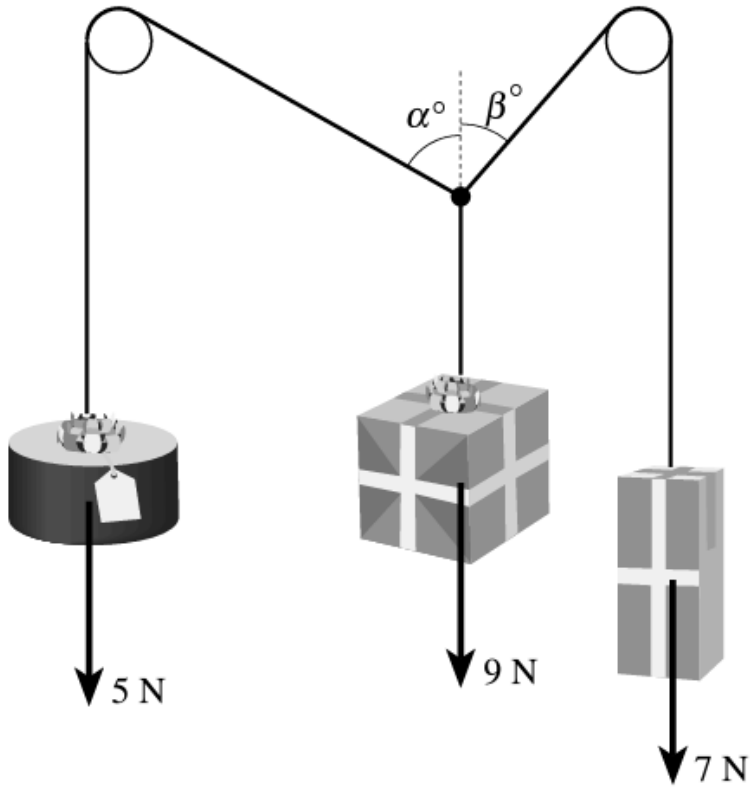
$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 5.14\ldots \\ 6.12\ldots \end{pmatrix} + \begin{pmatrix} 0 \\ -12 \end{pmatrix} + \begin{pmatrix} -9.39\ldots \\ 3.42\ldots \end{pmatrix}$$

$$= \begin{pmatrix} 6 + 5.14\ldots + 0 + (\pm 9.39\ldots) \\ 0 + 6.12\ldots + (-12) + 3.42\ldots \end{pmatrix} = \begin{pmatrix} 1.75 \\ -2.45 \end{pmatrix}, \text{ to 3 significant figures.}$$

10.5 Equilibrium

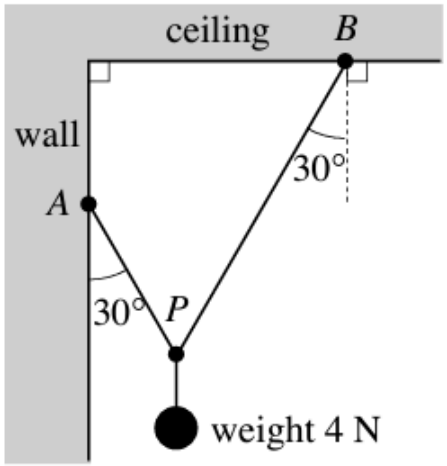
Example 10.5.1

Three strings are knotted together at one end, and parcels of weights 5 N , 7 N and 9 N are attached to the other ends. The first two strings are placed over smooth horizontal pegs, and the third parcel hangs freely, as shown in Fig. 10.28. The system is in equilibrium. Find the angles which the first two strings make with the vertical between the knot and the pegs.

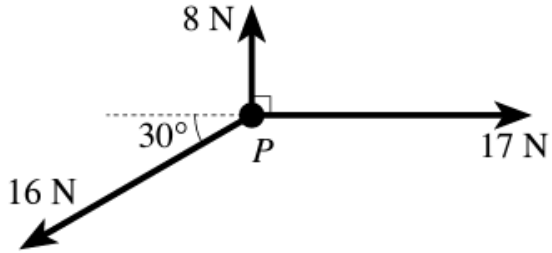


Class exercises
Miscellaneous Ex. 10 pg. 165

19 A small decoration is suspended by light strings AP and PB. The point A is on a vertical wall and the point B is on a ceiling. The strings AP and PB are at $30°$ to the vertical. The decoration, of weight 4 N, hangs in equilibrium from P. Show that there is the same tension in the strings AP and PB and calculate this tension. (MEI, adapted)



12 Three constant horizontal forces, with magnitudes and directions as shown in the diagram, act on a small block. Find the magnitude and direction of the resultant of these three forces.



The block is moving on a rough horizontal plane under the action of the three forces. Given that the mass of the block is $2\,\text{kg}$, and that its acceleration has magnitude $0.3\,\text{m s}^{-2}$, calculate the coefficient of friction between the block and the plane. (OCR)