11. General motion in a straight line

- Know how to use differentiation and integration to find expressions for acceleration, velocity and displacement as functions of time

11.1 Velocity and acceleration

What is acceleration?

It is the rate at which the velocity changes!

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

Example 11.1.1

A car starts to accelerate as soon as it leaves a town. After t seconds its velocity v m s^{-1} is given by the formula $v = 14 + 0.45t^2 - 0.03t^3$, until it reaches maximum velocity. Find a formula for the acceleration. How fast is the car moving when its acceleration becomes zero?

11.2 Displacement and velocity

What is velocity?

It is the rate at which the displacement changes!

$$v = \frac{\mathrm{d}x}{\mathrm{d}t}$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

Example 11.2.1

A space probe is launched by rockets. For the first stage of its ascent, which is in a vertical line and lasts for 40 seconds, the height x metres after t seconds is modelled by the equation $x = 50t^2 + \frac{1}{4}t^3$. How high is the probe at the end of the first stage, and how fast is it then moving?

For an object moving in a straight line, if x denotes the displacement from a fixed point O of the line at time t, v denotes the velocity and a the acceleration, then

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} \quad \text{and} \quad a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}.$$

The velocity is represented by the gradient of the (t,x) graph.

The acceleration is represented by the gradient of the (t,v) graph.

Example 11.2.2

A remote-controlled toy racing car moves along a straight track laid on the floor. It starts at a point O and for the next 6 seconds its displacement x cm is modelled by the formula $x = t^3(t-4)(t-7)$, where t is the time in seconds. Describe the motion of the car in detail.

Will get back to this

11.3 The reverse problem

From displacement to velocity: $$v = \frac{\mathrm{d}x}{\mathrm{d}t}$$

From velocity to acceleration: $$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

From acceleration to velocity? $$v = \int a dt + constant$$

From velocity to displacement? $$x = \int v dt + constant$$

Example 11.3.1

A train of mass 500 tonnes is travelling on a straight track at 48 m s^{-1} when the driver sees an amber light ahead. He applies the brakes for a period of 30 seconds with a force given by the formula $4t(30-t)$ kN, where t is the time in seconds after the brakes are applied. Find how fast the train is moving after 30 seconds, and how far it has travelled in that time.

Self study

From acceleration to velocity: $v = \int a dt + k$

From velocity to displacement: $x = \int v dt + k$

For an object moving in a straight line, with the velocity v given as a function of the time t, the displacement between times t_1 and t_2 is given by

$$\int_{t_1}^{t_2} v \, \mathrm{d}t.$$

This displacement is represented by the area under the (t, v) graph for the interval $t_1 \leqslant t \leqslant t_2$.

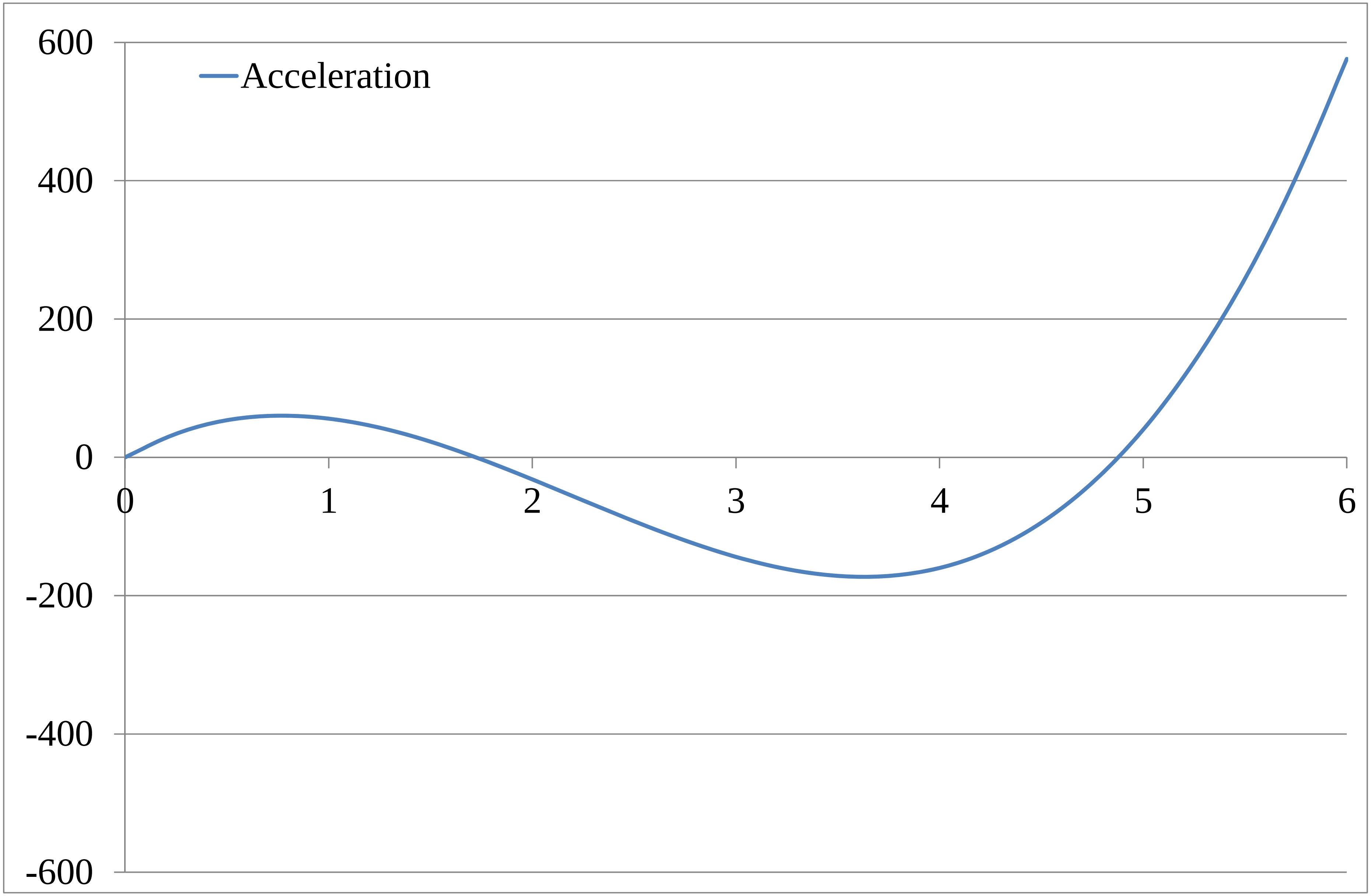
11.4 The constant acceleration formulae

From displacement to velocity: $$v = \frac{\mathrm{d}x}{\mathrm{d}t}$$

From velocity to acceleration: $$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

Example 11.2.2

A remote-controlled toy racing car moves along a straight track laid on the floor. It starts at a point O and for the next 6 seconds its displacement x cm is modelled by the formula $x = t^3(t-4)(t-7)$, where t is the time in seconds. Describe the motion of the car in detail.



600
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-200
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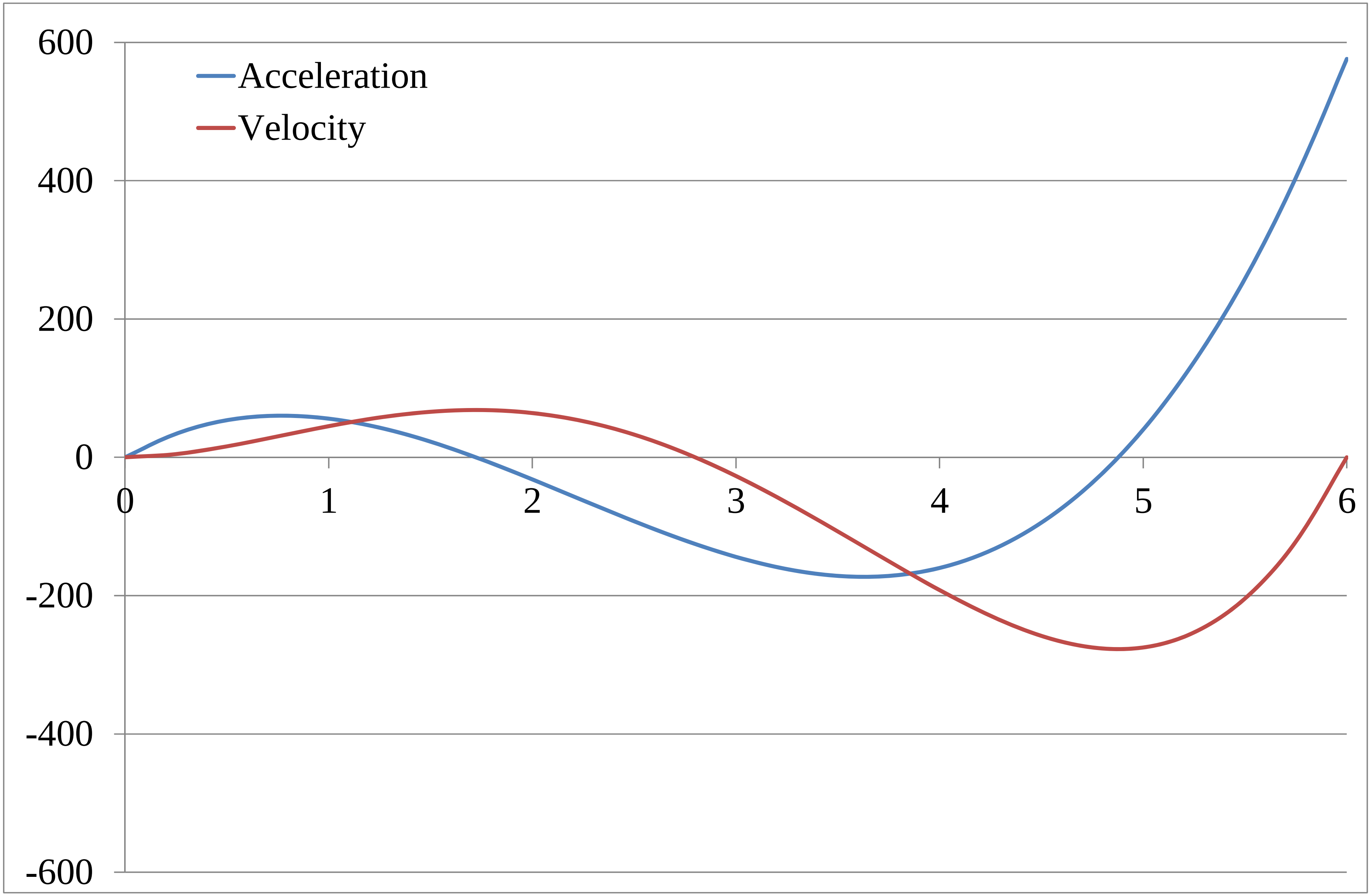
Acceleration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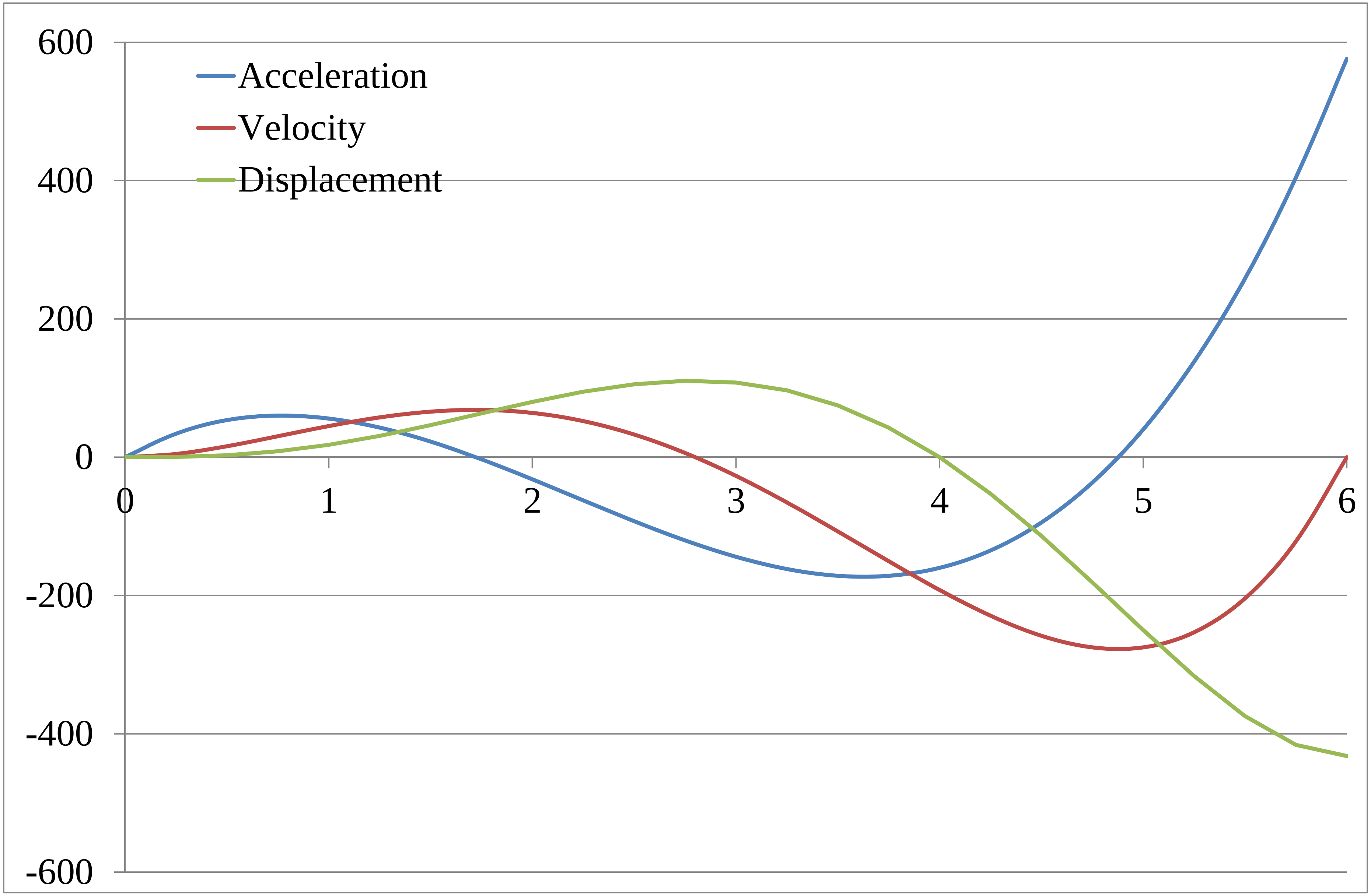
Acceleration
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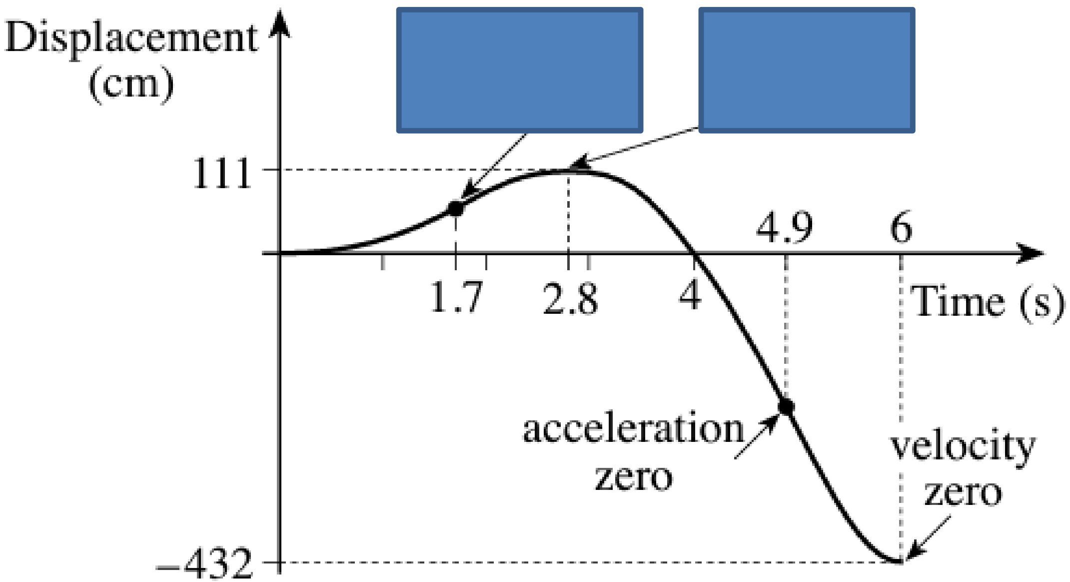


Fig. 11.1

$$v = \frac{\mathrm{d}x}{\mathrm{d}t}$$

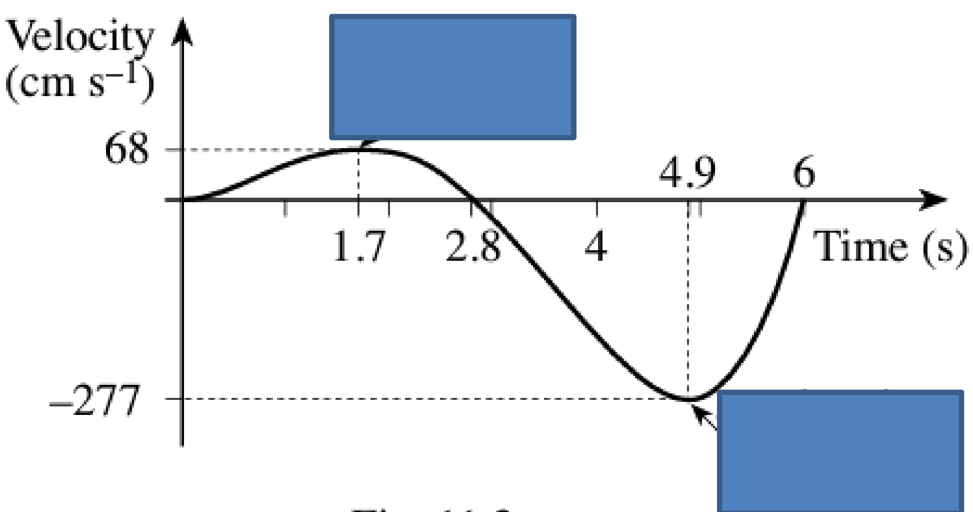


Fig. 11.2

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

Class exercises
Miscellaneous Exercise 11 pg. 181

1 A particle starts from O and moves along a straight line. At time t seconds its displacement from O is x cm and its velocity is $\left(10t - t^2\right)$ cm s^{-1}.

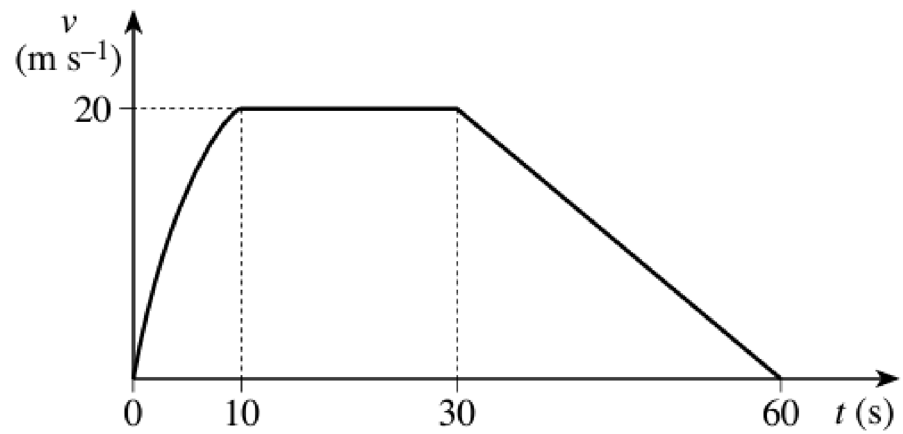
(a) Find x in terms of t.

(b) Find an expression for the acceleration of the particle in terms of t.

(c) Find the distance covered and the velocity at the moment when the acceleration is zero.

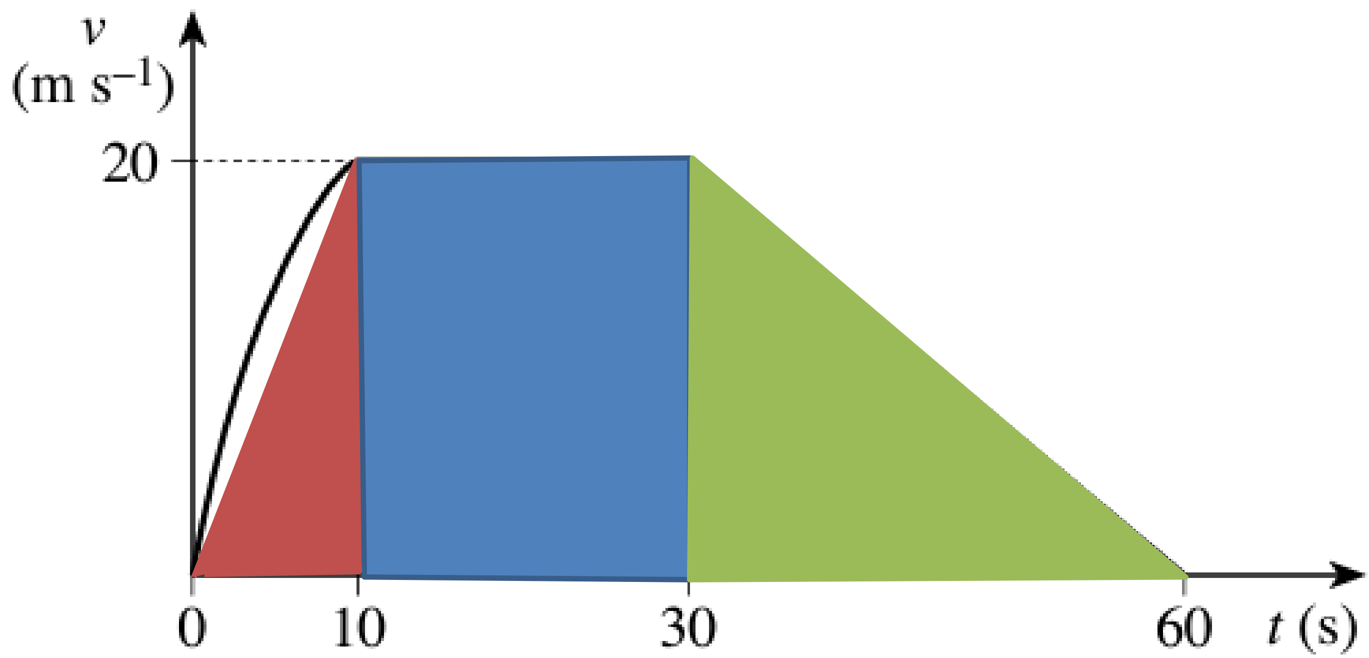
(d) Find the average velocity of the particle during the first 3 seconds. Show that this is less than the actual velocity after 1.5 seconds. (OCR)

4 The (t, v) graph for a motorcyclist travelling on a straight course is shown in the figure, for $0 \leqslant t \leqslant 60$, where t is the time measured in seconds and v is the velocity measured in m s^{-1}. Show that the total distance travelled during the 60 seconds is greater than 800 m.



Explain how you could use the (t, v) graph to estimate the time at which the motorcyclist was accelerating at 2 m s^{-2}. (OCR)

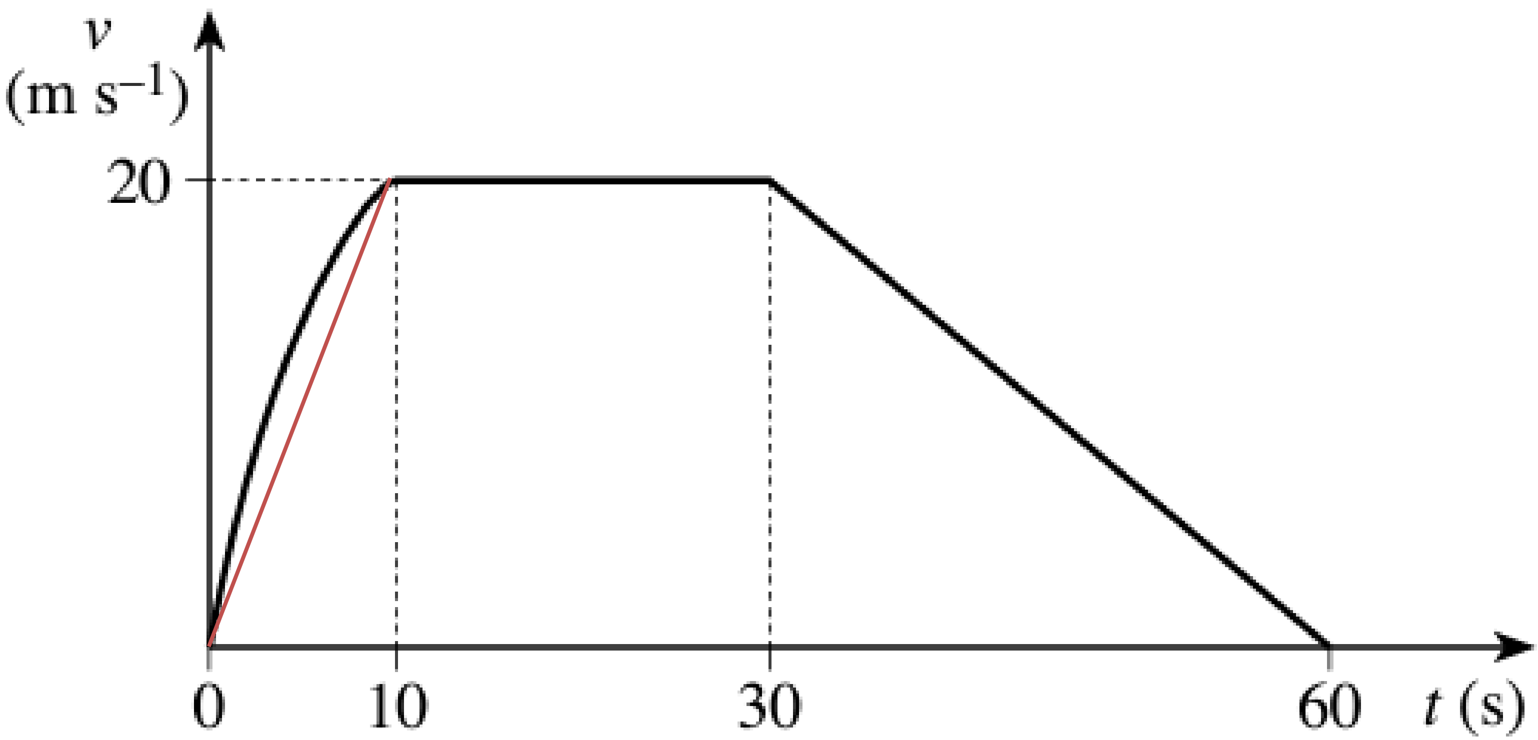
Distance travelled is area under (t, v) graph.



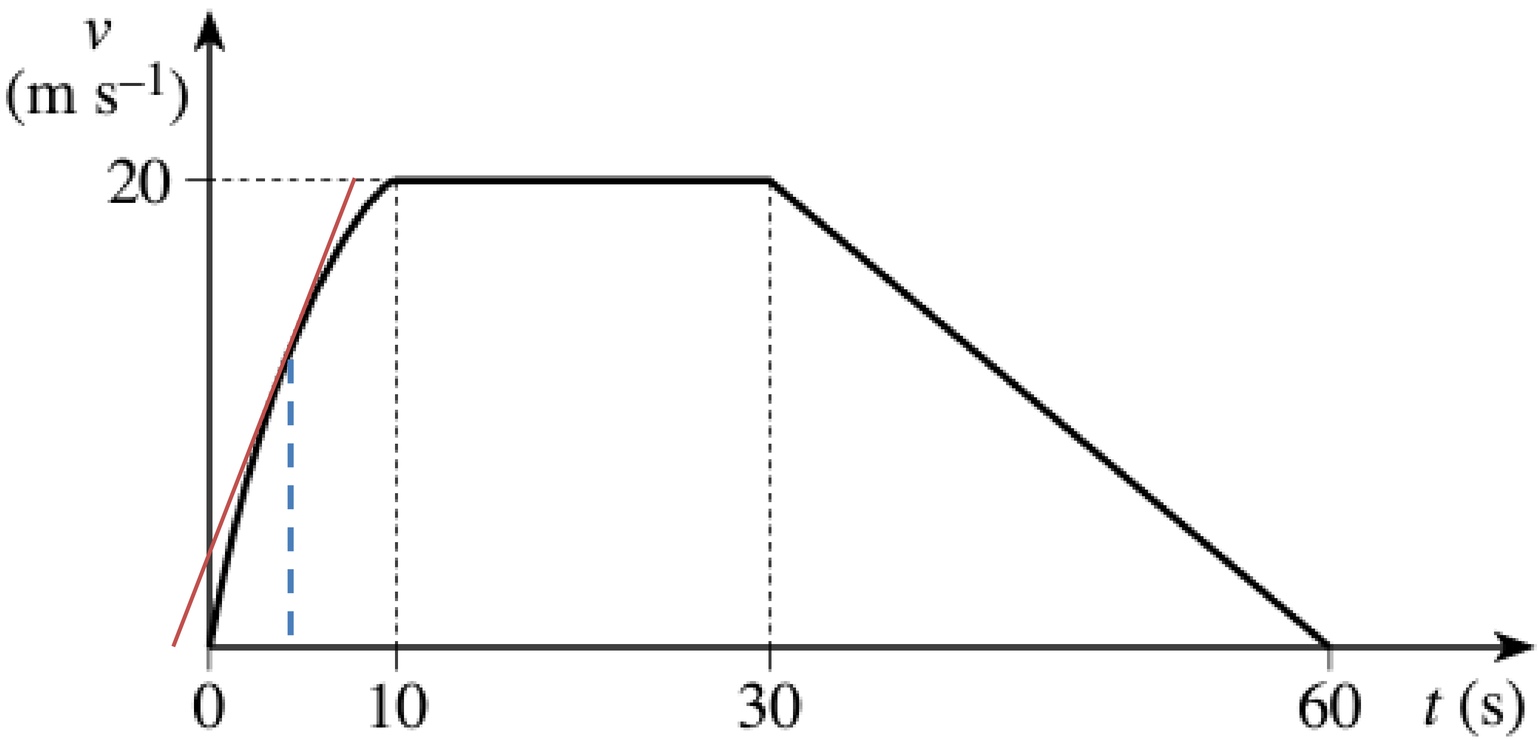
$Area_1 = 100$ $Area_2 = 400$ $Area_3 = 300$

Acceleration is tangent on (t, v) graph.

$$a = \frac{dv}{dt}$$



Acceleration is tangent on (t, v) graph.



$t \approx 4$ s

Homework assignment

Start working through past papers!