1. Velocity and acceleration (Kinematics)

- Displacement, velocity, acceleration and deceleration for motion in straight line
- Displacement-time and velocity-time graphs
- Speed in different systems of units
- Formulae for constant velocity and acceleration
- Problem solving: Constant velocity and acceleration

More equations for constant acceleration

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

Only if $v = u$ is $s = ut$

$$v^2 = u^2 + 2as$$

For an object moving with constant acceleration a and initial velocity u, the following equations connect the displacement s and the velocity v after a time t.

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t \qquad s = vt - \frac{1}{2}at^2$$

2. Force and motion

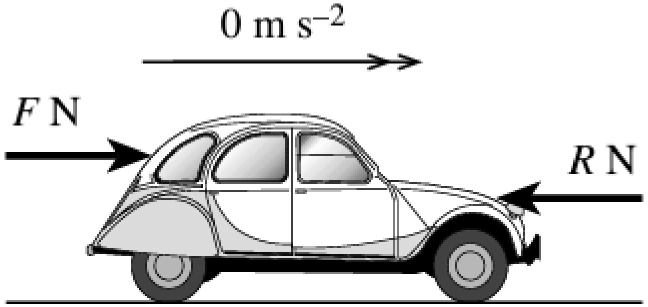
- Understand Newton's first law of motion
- Know some different types of forces
- Know and be able to apply Newton's second law to simple examples of objects moving in a straight line
- Understand the idea of equilibrium

Newton

Newton's first law Every object remains in a state of rest or of uniform motion in a straight line unless forces act on it to change that state.

Newton's second law When a force of F newtons acts on an object of mass m kg, it produces an acceleration, a m s^{-2}, given by $F = ma$.

2.4 Forces acting together



$$\overset{+}{\rightarrow}) \sum F = ma$$

$$\underbrace{F - R} = 0$$

If several forces act on an object parallel to a given direction, then the **net force** is the sum of the forces in that direction minus the sum of the forces in the opposite direction.

If the net force is zero, the forces on the object are said to be in **equilibrium**. The object then remains at rest, or moves with constant velocity. (Newton's first law)

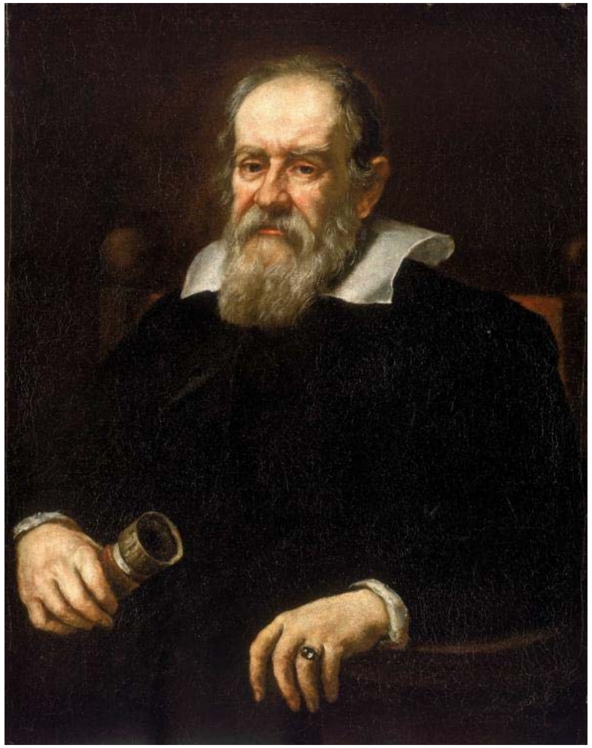
The net force is equal to the product of the mass of the object and its acceleration in the given direction. (Newton's second law)

Self study	**To be handed in**
Read: pp. 31-34 Examples: 3.2.1- 3.2.4 Exercise 3A: 1, 2, 4, 10, 15	Exercise 3A: 4, 10, 15

3. Vertical motion

- Know that if there is no air resistance, objects fall with a constant acceleration g
- Know the meaning of weight, and be able to distinguish weight from mass
- Know that an object of mass m has weight mg
- Be able to write equations for motion and equilibrium in a vertical direction
- Understand the normal contact force
- Understand the function of scales and balances for measuring mass

3.1 Acceleration due to gravity



Galileo Galilei
15 February 1564 – 8 January 1642

All objects, when dropped, fall towards the earth in a vertical line with the same constant acceleration, provided that there is no air resistance.

3.1 Acceleration due to gravity

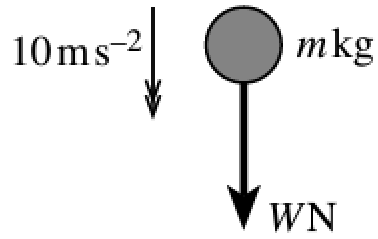
Acceleration due to gravity: $g = 9.81$ m s^{-2}

$g = 9.8$ m s^{-2}

We will use: $g = 10$ m s^{-2}

3.2 Weight

The **weight** of an object on or near the surface of the earth is the force of gravity with which the earth attracts it.



In SI units, the weight of an object of mass m kilograms is approximately $10m$ newtons. $=10m$ N

The weight of an object of mass m is mg.

Example 3.2.1

Determine the weight of

(a) a table of mass 42 kg, (b) a car of mass 1 tonne, (c) a sack of mass 15 lb

(a) Taking g as 10 m s^{-2}.

$$W = mg = 42(10) = 420 \text{ N}$$

(b) 1 tonne = 1000 kg

$$W = mg = 1(1000)(10) = 10000 \text{ N} = 10 \text{ kN}$$

(c) 1 kg is approximately 2.2 lb.

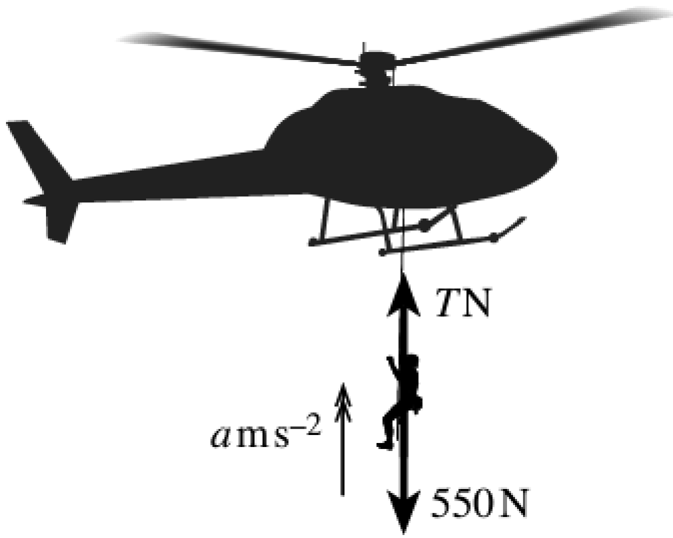
$$W = mg = \frac{15}{2.2}(10) = 68.0 \text{ N}$$

Example 3.2.2

An injured sailor is being winched up to a rescue helicopter. The mass of the sailor is 55 kg. Find the tension in the cable when the sailor is being raised

(a) at a steady speed of 4 m s^{-1},

(b) with an acceleration of 0.8 m s^{-2}.



(a) ↑) $F = ma$: $T - mg = 0$

$\therefore T - 550 = 0 \quad \Rightarrow T = 550\text{ N}$

(b) ↑) $F = ma$: $T - mg = ma$

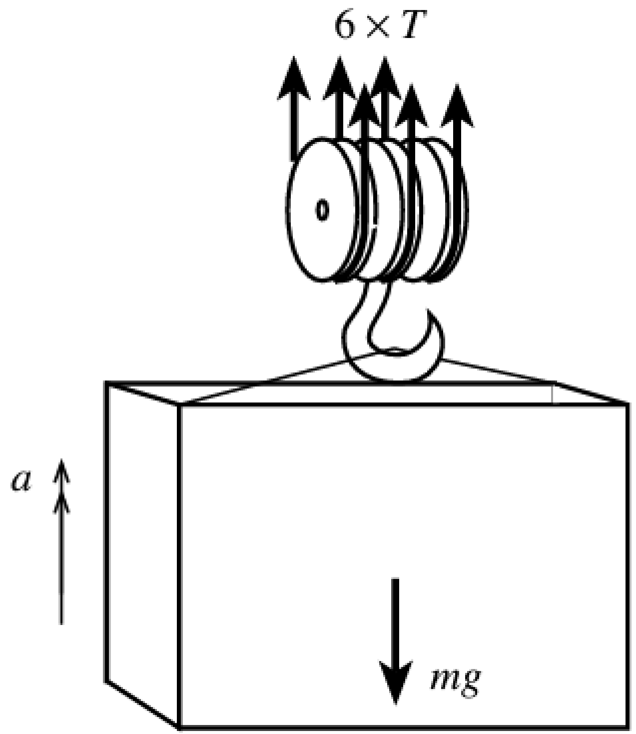
$\therefore T - 550 = 55(0.8)$

$\therefore T = 550 + 55(0.8)$

$\therefore T = 594\text{ N}$

Example 3.2.3

A pulley system is used to lift a heavy crate. There are six vertical sections of rope, each having tension T, and the crate has an upward acceleration a. Find the mass of the crate, expressing your answer in terms of T, a and g.



$\uparrow)\ F = ma$: $\quad 6T - mg = ma$

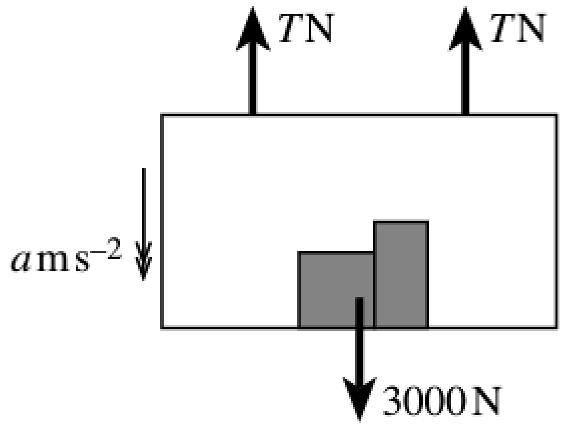
$$\therefore ma + mg = 6T$$

$$\therefore m(a + g) = 6T$$

$$\therefore m = \frac{6T}{a + g}$$

Example 3.2.4

Machinery of total mass 280 kg is being lowered to the bottom of a mine by means of two ropes attached to a cage of mass 20 kg. For the first 3 seconds of the descent, the tension in each rope is 900 N. Then for a further 16 seconds, the tension in each rope is 1500 N. For the final 8 seconds, the tension in each rope is 1725 N. Find the depth of the mine.



$\downarrow) \; F = ma: \qquad -2T + mg = ma$

<u>Stage 1:</u> $\quad T_1 = 900 \qquad \Delta t_1 = 3 \text{ s}$

$-2(900) + (280 + 20)(10) = (280 + 20)a_1$

$a_1 = 4 \text{ m/s}^2$ Downwards

$\downarrow) \; v_1 = u_1 + a_1 \Delta t_1$

$\therefore \; v_1 = 0 + 4(3) = 12 \text{ m/s}$

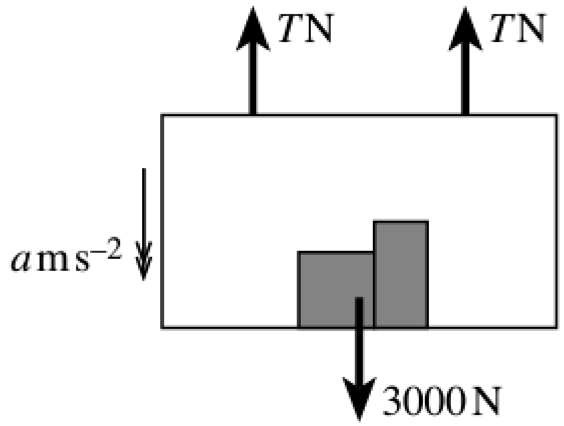
$\downarrow) \; s_1 = u_1 \Delta t_1 + \frac{1}{2} a_1 (\Delta t_1)^2 = 0 + \frac{1}{2}(4)(3)^2$

$\therefore s_1 = 18 \text{ m}$

Example 3.2.4

Machinery of total mass 280 kg is being lowered to the bottom of a mine by means of two ropes attached to a cage of mass 20 kg. For the first 3 seconds of the descent, the tension in each rope is 900 N. Then for a further 16 seconds, the tension in each rope is 1500 N. For the final 8 seconds, the tension in each rope is 1725 N. Find the depth of the mine.

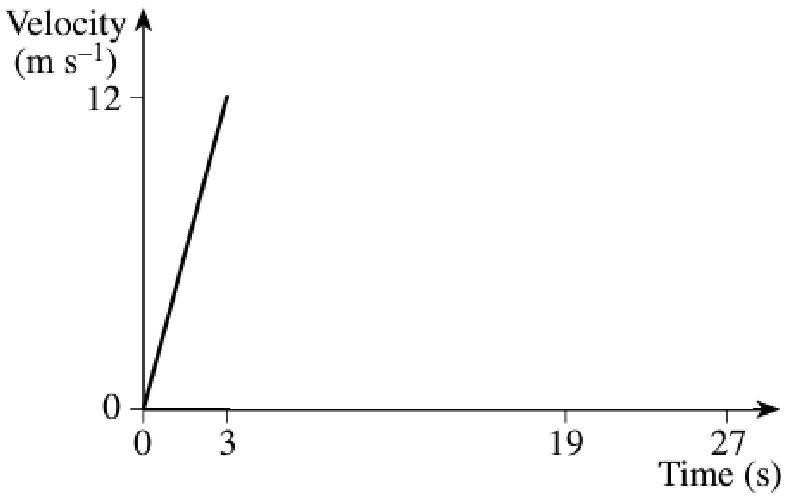
↓) $F = ma$: $-2T + mg = ma$



<u>Stage 1:</u> $a_1 = 4\text{ m/s}^2$ Downwards

$v_1 = 12\text{ m/s}$ Downwards

$s_1 = 18\text{ m}$



Example 3.2.4

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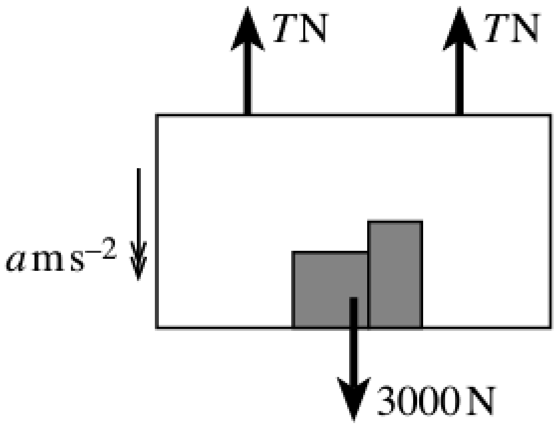
$\downarrow) \; F = ma: \qquad -2T + mg = ma$



<u>Stage 2:</u> $\qquad T_2 = 1500 \qquad \Delta t_2 = 16 \text{ s}$

$-2(1500) + (300)(10) = (300)a_2$

$a_2 = 0 \text{ m/s}^2$

$\downarrow) \; v_2 = u_2 + a_2 \Delta t_2 \quad = v_1 + 0$

$\therefore \; v_2 = v_1 = 12 \text{ m/s}$

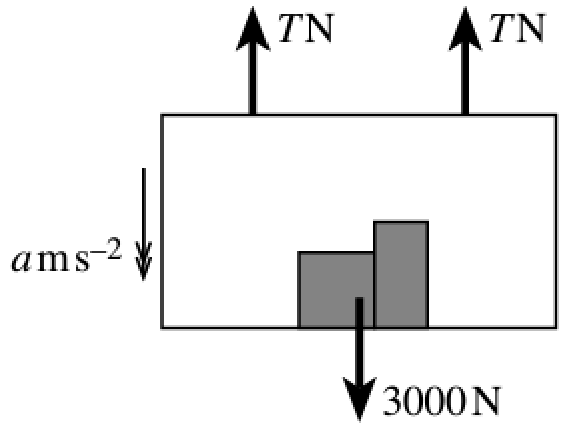
$\downarrow) \; s_2 = u_2 \Delta t_2 + \frac{1}{2} a_2 (\Delta t_2)^2 \quad = 12(16) + 0$

$\therefore \; s_2 = 192 \text{ m}$

Example 3.2.4

Machinery of total mass 280 kg is being lowered to the bottom of a mine by means of two ropes attached to a cage of mass 20 kg. For the first 3 seconds of the descent, the tension in each rope is 900 N. Then for a further 16 seconds, the tension in each rope is 1500 N. For the final 8 seconds, the tension in each rope is 1725 N. Find the depth of the mine.

$\downarrow)\ F = ma$: $\quad -2T + mg = ma$



<u>Stage 1:</u> $a_1 = 4\ \text{m/s}^2$ Downwards

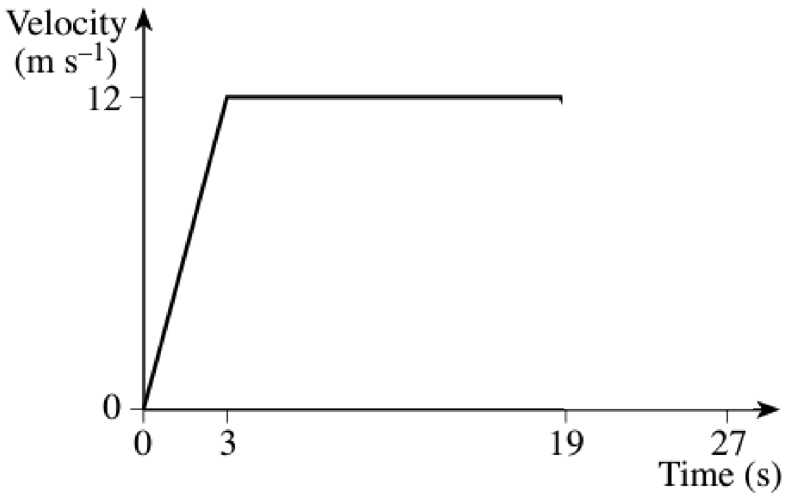
$v_1 = 12\ \text{m/s}$ Downwards

$s_1 = 18\ \text{m}$

<u>Stage 2:</u> $a_2 = 0\ \text{m/s}^2$

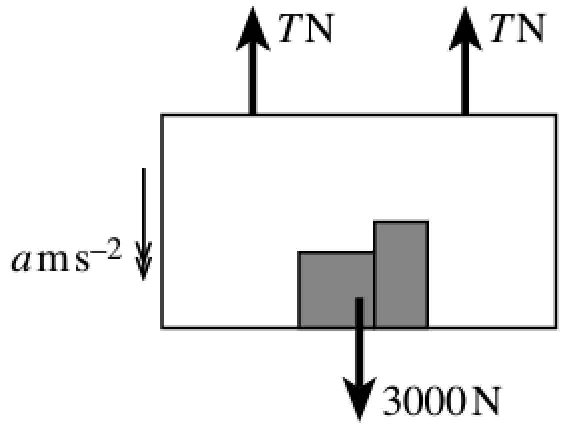
$v_2 = 12\ \text{m/s}$ Downwards

$s_2 = 192\text{m}$



Example 3.2.4

Machinery of total mass 280 kg is being lowered to the bottom of a mine by means of two ropes attached to a cage of mass 20 kg. For the first 3 seconds of the descent, the tension in each rope is 900 N. Then for a further 16 seconds, the tension in each rope is 1500 N. For the final 8 seconds, the tension in each rope is 1725 N. Find the depth of the mine.



$\downarrow$) $F = ma$: $\quad -2T + mg = ma$

Stage 3: $\quad T_3 = 1725 \quad \Delta t_3 = 8$ s

$-2(1725) + (300)(10) = (300)a_3$

$a_3 = -1.5 \text{ m/s}^2$

$\therefore a_3 = 1.5 \text{ m/s}^2$ upwards

$\downarrow$) $v_3 = u_3 + a_3\Delta t_3 \quad = 12 + (-1.5)(8)$

$\therefore \; v_3 = 0$

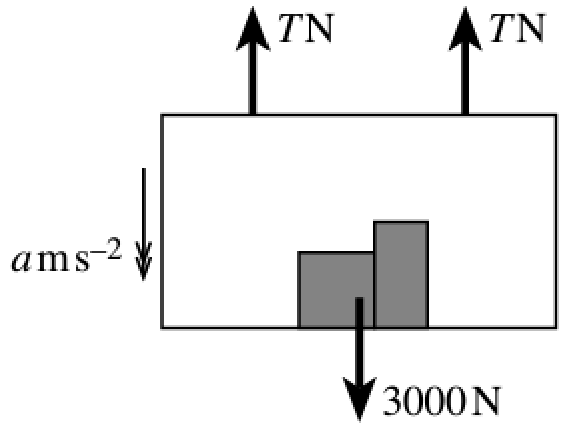
$\downarrow$) $s_3 = u_3\Delta t_3 + \frac{1}{2}a_3(\Delta t_3)^2 \; = 12(8) + \frac{1}{2}(-1.5)(8)$

$\therefore s_3 = 48$ m

Example 3.2.4

Machinery of total mass 280 kg is being lowered to the bottom of a mine by means of two ropes attached to a cage of mass 20 kg. For the first 3 seconds of the descent, the tension in each rope is 900 N. Then for a further 16 seconds, the tension in each rope is 1500 N. For the final 8 seconds, the tension in each rope is 1725 N. Find the depth of the mine.

$\downarrow) \ F = ma$: $\quad -2T + mg = ma$



<u>Stage 1:</u> $a_1 = 4 \text{ m/s}^2$ Downwards

$v_1 = 12 \text{ m/s}$ Downwards

$s_1 = 18 \text{ m}$

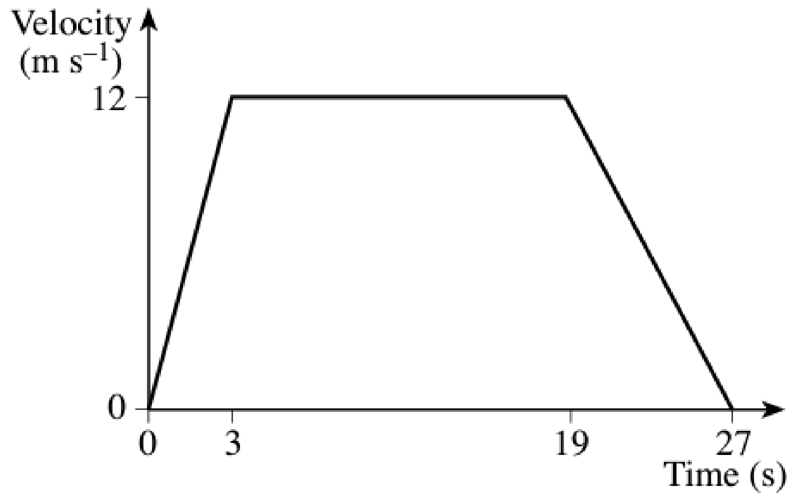
<u>Stage 2:</u> $a_2 = 0 \text{ m/s}^2$

$v_2 = 12 \text{ m/s}$ Downwards

$s_2 = 192\text{m}$

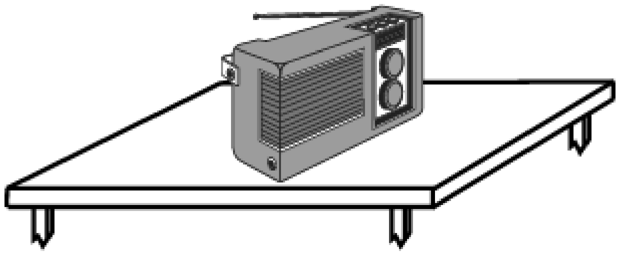
<u>Stage 3:</u> $a_3 = 1.5 \text{ m/s}^2$ Upwards

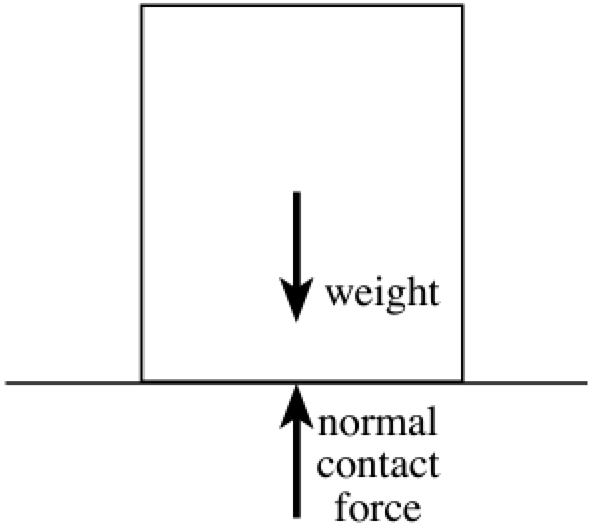
$v_3 = 0 \qquad s_3 = 48\text{m}$

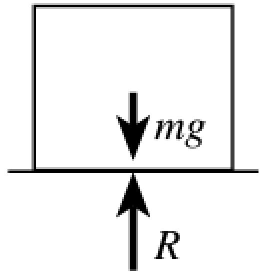


$\therefore s_{total} = 18 + 192 + 48 = 258 \text{ m}$

3.3 Normal contact force



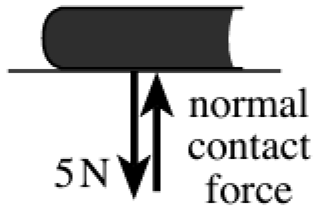




When an object is in contact with a surface, there is a force on the object at right angles to the region of contact. This is called the **normal contact force**.

Example 3.3.1

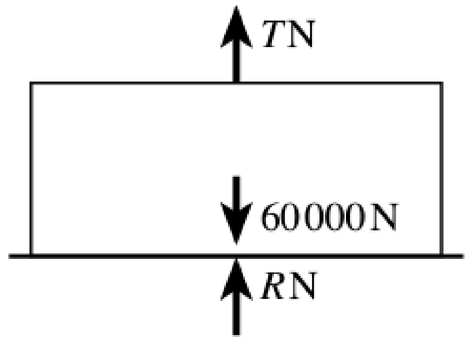
A book of mass $0.5\ \text{kg}$ is placed flat on a horizontal shelf, as in Fig. 3.11. Find the magnitude of the normal contact force.



Self study

Example 3.3.2

A container sits on the dockside waiting to be loaded on to a container ship. The mass of the container is 6000 kg . A cable from a crane is attached to the container. At first, the cable is slack; the tension is then gradually increased until the container rises off the ground. Draw a graph to show the relationship between the normal contact force and the tension in the cable.

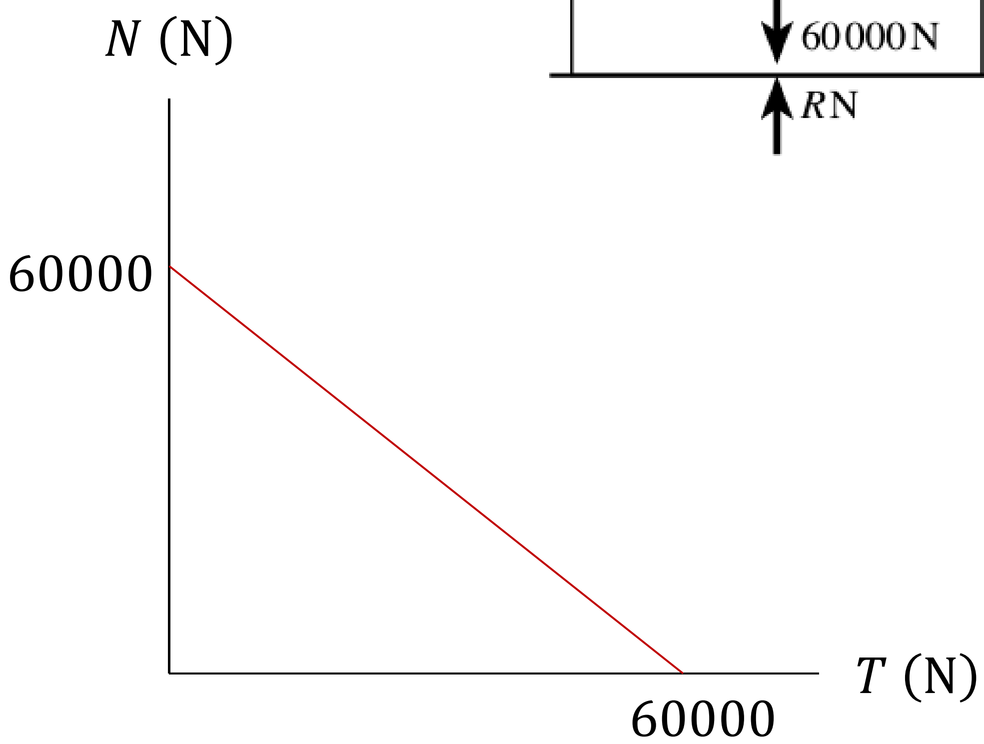


$\uparrow)\ F = ma$:

$$T + N - mg = 0$$

$$\therefore T + N - 6000(10) = 0$$

$$\therefore N = -T + 60000$$



3.4 Mass and weight

Here are some remarks which you might hear or read in a newspaper.

'This bag of potatoes has a weight of 3 kilograms.'
'The weight of an elephant is about 6 tonnes.'
'My rucksack weighs 16 kg.'
'The boxer weighed in at 159 pounds.'

These are all statements for which, in mechanics, the correct usage would be to replace 'weight' by 'mass', and 'weighs' by 'has mass'.

In the case of the potatoes, you might be interested in both the mass and the weight. Mass is important from the point of view of feeding the family; but if you have to carry the bag home, it is the weight of about 30 newtons that will concern you, since this has to be supported by your arm muscles.

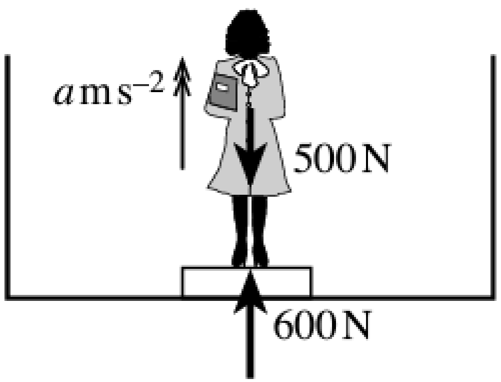
With the elephant, it is mass which matters if it charges your safari jeep; but if it walks across a bridge, it is the weight which might cause the bridge to collapse.

The mass of the rucksack remains constant, but its weight (about 160 newtons) decreases very slightly as you climb a mountain, because the value of g decreases with height.

For the boxer, the weight is irrelevant. It is his mass which determines how he can stand up to a punch, or how much he can damage his opponent.

Example 3.4.1

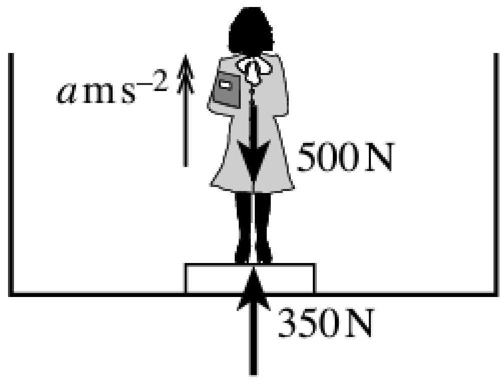
A mechanics student lives on the tenth floor of a tall building. She has just bought new bathroom scales, and decides to try them out by standing on them as she goes up in the lift. Initially the scales read 50 kg. After the doors have closed the reading briefly goes up to 60 kg, but then returns to 50 kg. As the lift nears the tenth floor, the reading drops to 35 kg. Explain.



$\uparrow)\ F = ma$:

$600 - 500 = 50(a)$

$\therefore a = 2\ \mathrm{m/s^2}$ Upwards



$\uparrow)\ F = ma$:

$350 - 500 = 50(a)$

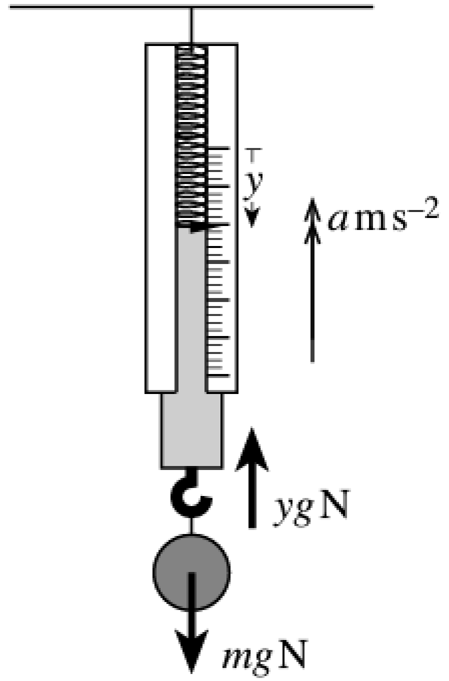
$\therefore a = -3\ \mathrm{m/s^2}$

$\therefore a = 3\ \mathrm{m/s^2}$ Downwards

Example 3.4.2

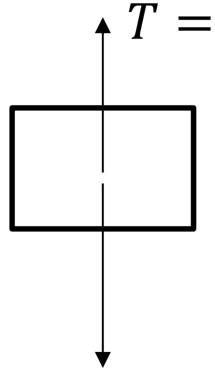
A heavy mass m kg is suspended from the roof of a lift by a wire. The wire is cut, and a spring balance is inserted between the two free ends. When the lift is accelerating upwards at $a \text{ m s}^{-2}$, the reading on the balance is y kg. Find the equation connecting a and y.

Self study



Class exercises: Exercise 3A

3 A crane is lifting a load of mass 350 kg. The tension in the cable as the load is lifted is 4200 N. Calculate the acceleration of the load.



$\uparrow)\ F = ma$: $\quad T - mg = ma$

$\therefore 4200 - 350(10) = 350a$

$\therefore a = 2\ \mathrm{m/s^2}$ Upwards

Class exercises: Exercise 3A

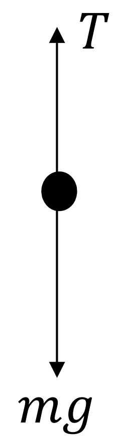
8 A boy of mass 45 kg is stranded on a beach as the tide comes in. A rescuer of mass 75 kg is lowered down, by rope, from the top of the cliff. They are raised together, initially with a constant acceleration of 0.6 m s^{-2}. Find the tension in the rope for this stage of the ascent.

As they near the top of the cliff, the tension in the rope is 1020 N and they are moving with a constant deceleration. Calculate the magnitude of this deceleration.

$\uparrow)\ F = ma$: $\quad T - mg = ma$

$\therefore T - (45 + 75)(10) = (45 + 75)(0.6)$

$\therefore T = 1272 \text{ N}$



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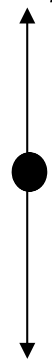
As they near the top of the cliff, the tension in the rope is 1020 N and they are moving with a constant deceleration. Calculate the magnitude of this deceleration.

$\uparrow)\ F = ma$: $\quad T - mg = ma$

$\therefore T - (45 + 75)(10) = (45 + 75)(0.6)$

$\therefore T = 1272 \text{ N}$



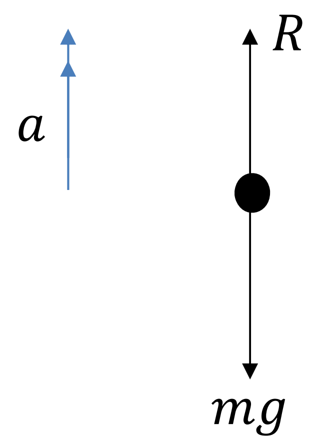
$\uparrow)\ F = ma$: $\quad T - mg = ma$

$\therefore 1020 - (45 + 75)(10) = (45 + 75)(a)$

$\therefore a = -1.5 \text{ m/s}^2$

$\therefore a = 1.5 \text{ m/s}^2$ Downwards

12 The resisting force, R N, experienced by a parachutist travelling with speed v m s^{-1} may be modelled as $R = 135v$. It may be assumed that the parachutist moves vertically downwards at all times. At the instant that she is moving with a speed of 8 m s^{-1} she has a deceleration of 2 m s^{-2}. Find her mass. The speed of the parachutist continues to drop until it reaches a constant value (the terminal speed); find this speed.



$\downarrow) \; F = ma: \quad mg - R = ma$

$\therefore m(10) - 135v = m(-2)$

If $v = 8$ m/s: $\quad m(10) - 135(8) = m(-2)$

$\therefore m = 90$ kg

$\downarrow) \; F = ma: \quad mg - R = 0$

$\therefore 90(10) - 135v = 0$

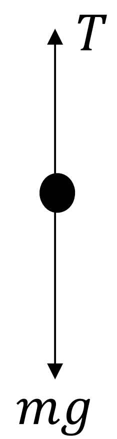
$\therefore v = 6.67$ m/s

14 A bucket of mass 4 kg is being lowered down a well at constant speed. Find the tension in the lowering rope. When filled with water, the bucket is raised with a constant acceleration of 0.8 m s^{-2} for part of the ascent. The tension in the rope in this stage is 216 N. Calculate the mass of water in the bucket.

$\downarrow$) $F = ma$: $\quad mg - T = 0$

$\therefore 4(10) - T = 0$

$\therefore T = 40 \text{ N}$ Upwards



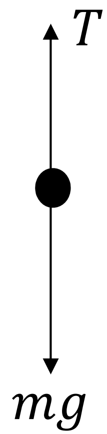
$\uparrow$) $F = ma$: $\quad T - mg = ma$

$\therefore 216 - (4 + m_w)(10) = (4 + m_w)0.8$

$\therefore m_w = 16 \text{ kg}$

17 A load of weight 7 kN is being raised from rest with constant acceleration by a cable. After the load has been raised 20 metres, the cable suddenly becomes slack. The load continues upwards for a distance of 4 metres before coming to instantaneous rest. Assuming no air resistance, find the tension in the cable before it became slack.

$$\uparrow)\ F = ma: \quad T - mg = ma$$



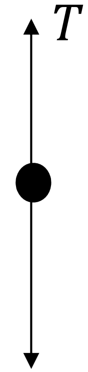
$$\therefore T - 7(1000) = \frac{7(1000)}{10}a$$

$$\therefore a_1 = \frac{(T - 7000)}{700}$$

$$v_1^2 = u_1^2 + 2a_1 s_1 \ \Rightarrow v_1^2 = 0 + 2\left(\frac{(T - 7000)}{700}\right)20$$

17 A load of weight 7 kN is being raised from rest with constant acceleration by a cable. After the load has been raised 20 metres, the cable suddenly becomes slack. The load continues upwards for a distance of 4 metres before coming to instantaneous rest. Assuming no air resistance, find the tension in the cable before it became slack.

$$v_1^2 = 2\left(\frac{(T-7000)}{700}\right)20 \qquad --(1)$$



$$v_2^2 = u_2^2 + 2a_2 s_2$$

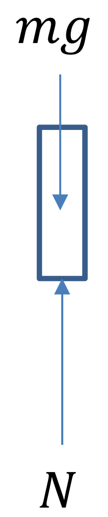
↑) Further: $v_2 = 0 \quad s_2 = 4 \quad a_2 = -10$

$$u_2^2 = v_1^2 = 2\left(\frac{(T-7000)}{700}\right)20$$

$$\therefore 0 = 2\left(\frac{(T-7000)}{700}\right)20 + 2(-10)(4)$$

$$\therefore T = 8400 \text{ N}$$

A boy of mass 60 kg is standing in a lift that has an upward acceleration of magnitude 0.50 m s^{-2}. Describe the forces acting on the boy, and find their magnitudes.



$\uparrow)\ F = ma$: $\quad N - mg = ma$

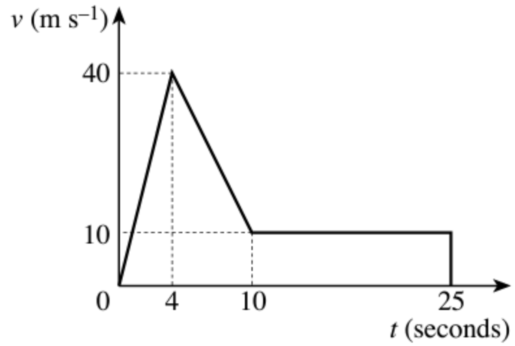
$\therefore N - 60(10) = 60(0.5)$

$\therefore N - 60(10) = 60(0.5)$

$\therefore N = 630 \text{ N}$

weight $= 600$ N

The diagram shows an approximate (t, v) graph for the motion of a parachutist falling vertically; v m s^{-1} is the parachutist's downwards velocity at time t seconds after he jumps out of the plane. Use the information in the diagram

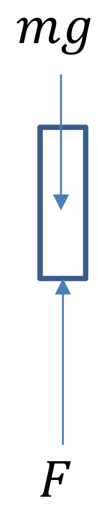


(a) to give a brief description of the parachutist's motion throughout the descent,

(b) to calculate the height from which the jump was made.

The mass of the parachutist is 90 kg. Calculate the upwards force acting on the parachutist, due to the parachute, when $t = 7$.

State two ways in which you would expect an accurate (t, v) graph for the parachutist's motion to differ from the approximate graph shown in the diagram. (OCR)



(a) Accelerates at 10 m/s^2 for 4 s; decelerates at 5 m/s^2 for 6 s; travel at constant speed of 10 m/s for 15 s.

Height is area under $v - t$ graph: $H = \frac{1}{2}4(40) + \frac{1}{2}6(30) + 21(10) = 380$ m

At $t = 7$ s: $a = -5$ m/s^2

$\uparrow)\ F = ma$: $F - mg = ma$ $\quad F - 90(10) = (10)(-5)$

$F = 850$ N

(1) A heavy ball is thrown straight down from a tower with an initial velocity of 50 m/s. ($g = 10\ \mathrm{m/s^2}$ down). After 2.0 s the magnitude of its velocity is:

A) 2.5 m/s

B) 30 m/s

C) 62 m/s

D) 110 m/s

E) 70 m/s

(2) A baseball is thrown vertically upward into the air. What is the instantaneous acceleration of the ball at its highest point?

A. 10 m/s^2 up.

B. zero.

C. 10 m/s^2 down.

D. changing from 10 m/s^2 up to 10 m/s^2 down.