Self study

Self study	**To be handed in**
Read: pp. 66-71 Examples: 5.5.2, 5.5.3 Exercise 5B: 1, 8, 14	Exercise 5B: 1, 8, 14

1. Equations for constant acceleration

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

Only if $v = u$ is $s = ut$

For an object moving with constant acceleration a and initial velocity u, the following equations connect the displacement s and the velocity v after a time t.

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t \qquad s = vt - \frac{1}{2}at^2$$

2. Force and motion

- Understand Newton's first law of motion
- Know some different types of forces
- Know and be able to apply Newton's second law to simple examples of objects moving in a straight line
- Understand the idea of equilibrium

2. Newton's laws

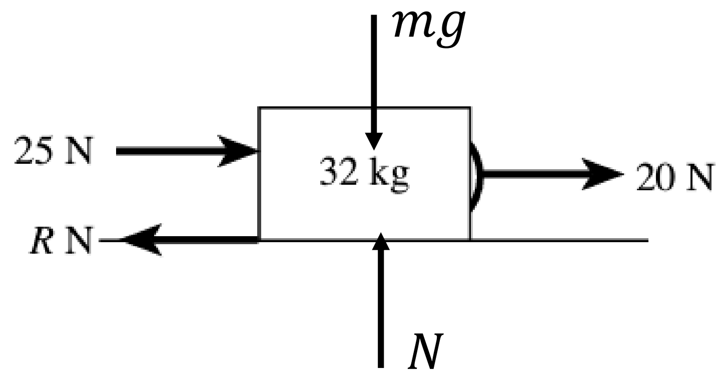
Newton's first law Every object remains in a state of rest or of uniform motion in a straight line unless forces act on it to change that state.

Newton's second law When a force of F newtons acts on an object of mass m kg, it produces an acceleration, a m s^{-2}, given by $F = ma$.

2. Newton's laws

Example 2.4.1

A heavy box of mass 32 kg has a handle on one side. Two children try to move it across the floor. One pulls horizontally on the handle with a force of 20 N, the other pushes from the other side of the box with a force of 25 N, but the box does not move. Find the frictional force resisting the motion.



$$\overset{+}{\rightarrow}) \sum F = ma$$

3. Vertical motion

- Know that if there is no air resistance, objects fall with a constant acceleration g
- Know the meaning of weight, and be able to distinguish weight from mass
- Know that an object of mass m has weight mg
- Be able to write equations for motion and equilibrium in a vertical direction
- Understand the normal contact force
- Understand the function of scales and balances for measuring mass

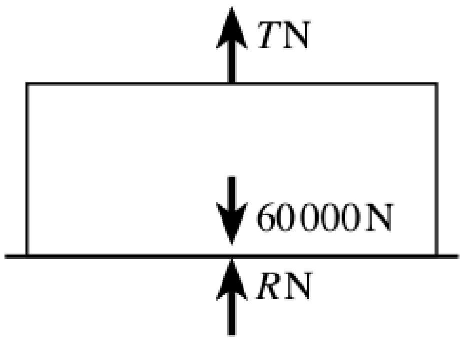
3 Normal contact force

When an object is in contact with a surface, there is a force on the object at right angles to the region of contact. This is called the **normal contact force**.

Example 3.3.2

A container sits on the dockside waiting to be loaded on to a container ship. The mass of the container is 6000 kg . A cable from a crane is attached to the container. At first, the cable is slack; the tension is then gradually increased until the container rises off the ground. Draw a graph to show the relationship between the normal contact force and the tension in the cable.

$g = 10 \text{ m s}^{-2}$



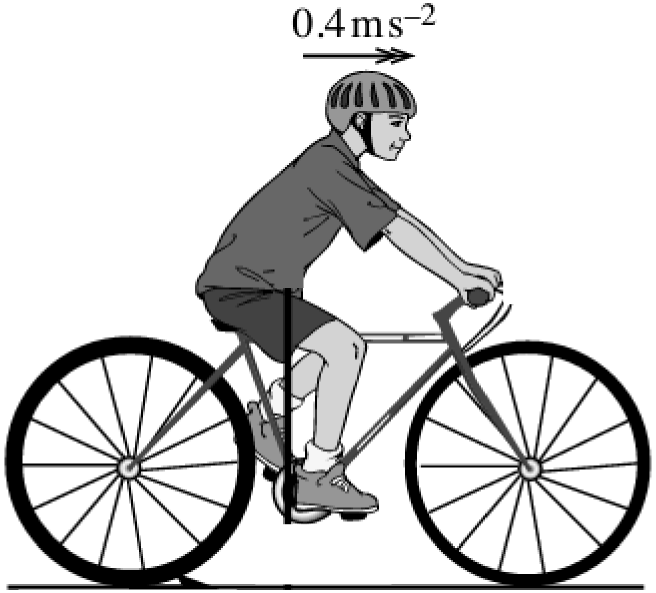
4. Resolving forces

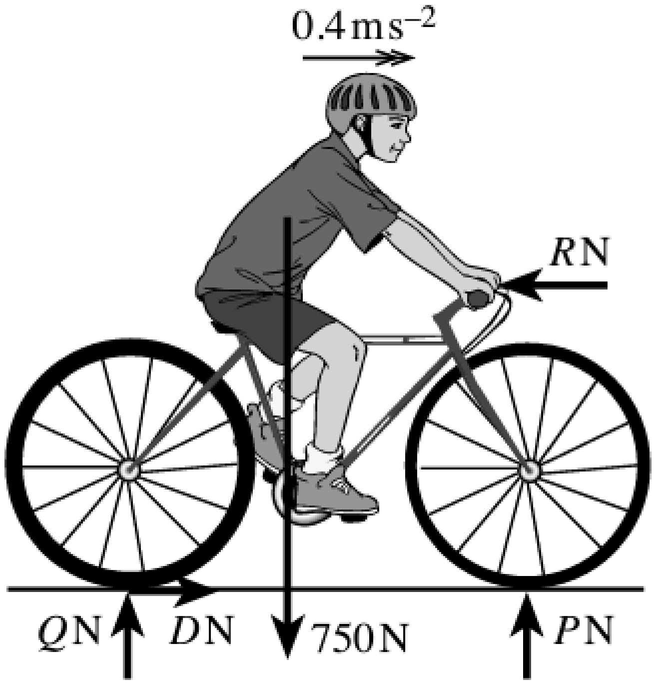
- Understand the idea of resolving in a chosen direction
- Know how to find the resolved part of a force in a given direction
- Able to solve problems by resolving in various directions

4.1 Resolving horizontally and vertically

Suppose that a cyclist starts from rest and accelerates at $0.4\ \text{m s}^{-2}$. The combined mass of the cyclist and machine is $75\ \text{kg}$.

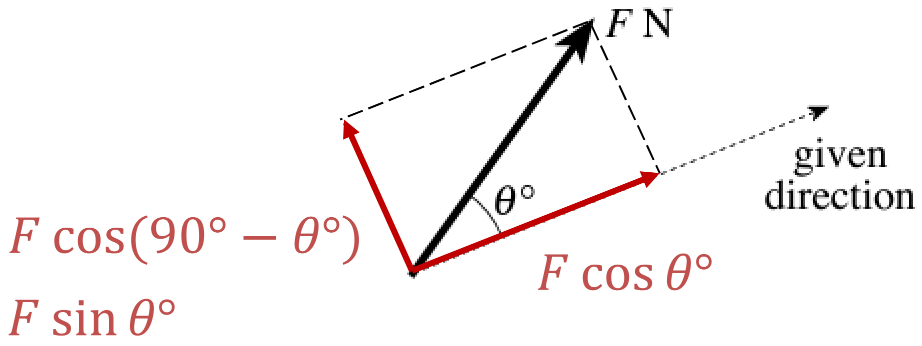
Since the cyclist and the bicycle have the same acceleration, they can be treated as a single object. What are the forces acting on it?





4.2 Forces at an angle

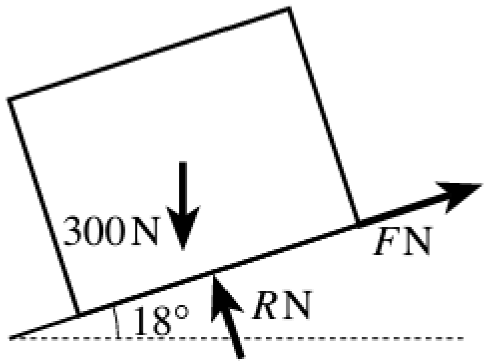
If a force of F newtons makes an angle of $\theta°$ with a given direction, then the effect of the force in that direction is $F \times \cos\theta°$. This is called the **resolved part** of the force in the given direction.

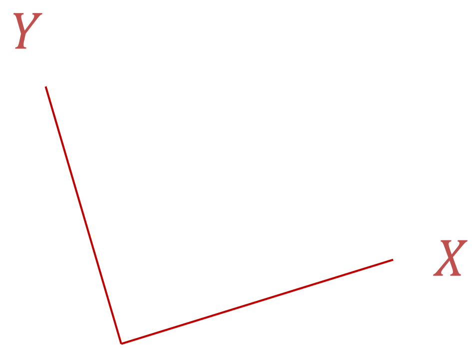


4.4 resolving in other directions

Example 4.4.1

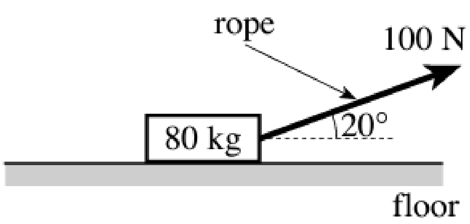
A crate of mass 30 kg is at rest on a ramp which slopes at an angle of 18° to the horizontal. It is prevented from sliding down the ramp by friction. Find the frictional force and the normal contact force.





Miscellaneous exercise 4

12 A box of mass 80 kg is to be pulled along a horizontal floor by means of a light rope. The rope is pulled with a force of 100 N and the rope is inclined at 20° to the horizontal, as shown in the figure.



(a) Explain briefly why the box cannot be in equilibrium if the floor is smooth.

In fact the floor is not smooth and the box is in equilibrium.

(b) Draw a diagram showing all the external forces acting on the box.

(c) Calculate the frictional force between the box and the floor, and also the normal reaction of the floor on the box, giving your answers correct to 3 significant figures.

The maximum value of the frictional force between the box and the floor is 120 N and the box is now pulled along the floor with the rope always inclined at 20° to the horizontal.

(d) Calculate the force with which the rope must be pulled for the box to move at a constant speed. Give your answer correct to 3 significant figures.

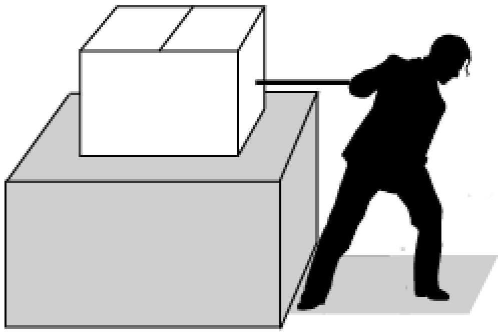
(e) Calculate the acceleration of the box if the rope is pulled with a force of 140 N.

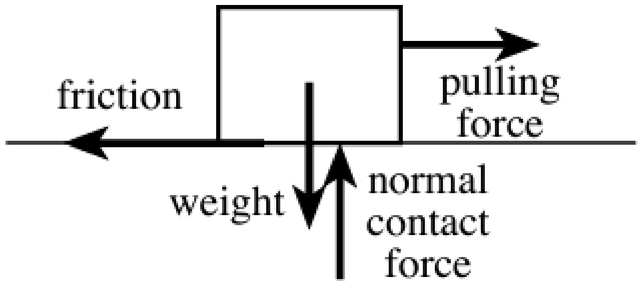
5. Friction

5. Friction

- Be familiar with the mathematical model of friction and the properties of frictional forces
- Understand the idea of limiting equilibrium
- Know what is meant by the coefficient of friction and be able to use it
- Be able to solve problems on motion and equilibrium in which friction is one of the forces actin on an object

5.1 Basic properties of frictional forces





When part of the surface of an object is in contact with a fixed surface, and forces are tending to move the object across the surface, these forces will be opposed by a frictional force. Its direction is opposite to the direction of motion or possible motion.

The frictional force cannot exceed a certain magnitude, called the **limiting friction**. If the object is at rest and equilibrium is possible with a frictional force less than this limiting friction, the object will remain in equilibrium.

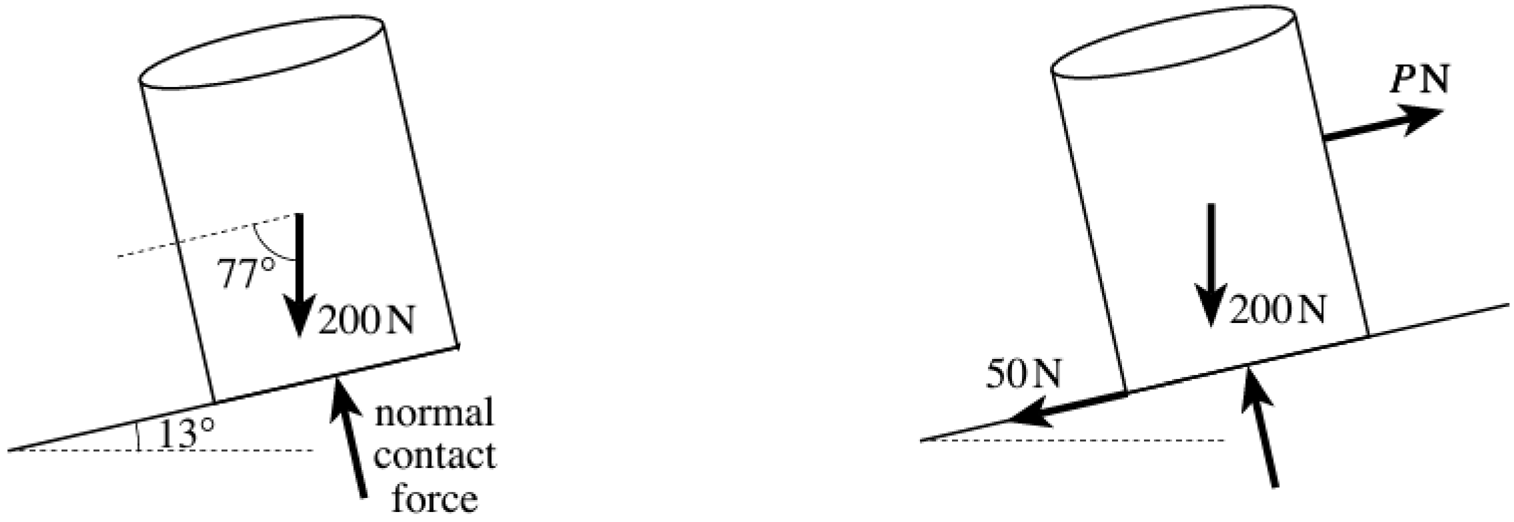
If the object is at rest and the forces are in equilibrium with the limiting friction, the object is said to be in **limiting equilibrium**, and to be 'on the point of moving'.

Example 5.1.1

A dustbin of mass 20 kg is placed on a path which is at an angle of 13° to the horizontal. The limiting friction between the bin and the path is 50 N .

(a) Will the bin slide down the path?

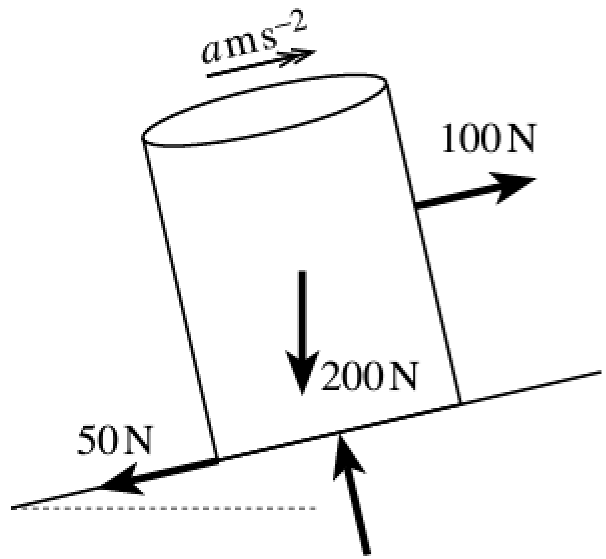
(b) A force parallel to the slope is applied to the bin so that it is on the point of moving up the path. How large is this force?



When an object slides over a fixed surface, the frictional force has its limiting value and acts in a direction opposite to the direction of motion.

Example 5.1.2

If, with the data in Example 5.1.1, a force of $100\ \text{N}$ is applied to the dustbin up the path, calculate the acceleration with which the bin will move.



5.2 Limiting friction

The next question to ask is, how large is the limiting friction?

This might depend on a number of factors, such as

- the materials that the surfaces in contact consist of,
- the shape and area of the region of contact between the surfaces,
- the other forces acting on the object.

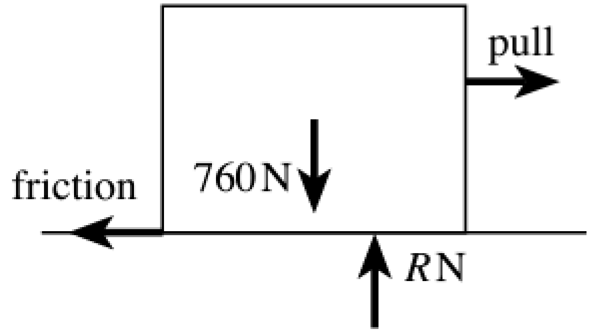
The limiting frictional force between two surfaces is proportional to the normal contact force. If the limiting friction is F_{lim} and the normal contact force is R, then $F_{\text{lim}} = \mu R$, where μ is a constant.

The constant μ is called the **coefficient of friction**. Its value depends mainly on the materials of which the surfaces consist.

Smooth means friction can be ignored (is negligible)

Example 5.2.1

A person tries to pull a small cupboard across the floor. The mass of the cupboard is 76 kg and the coefficient of friction is 0.5. Describe what happens if the cupboard is pulled with a horizontal force of (a) 200 N, (b) 400 N.



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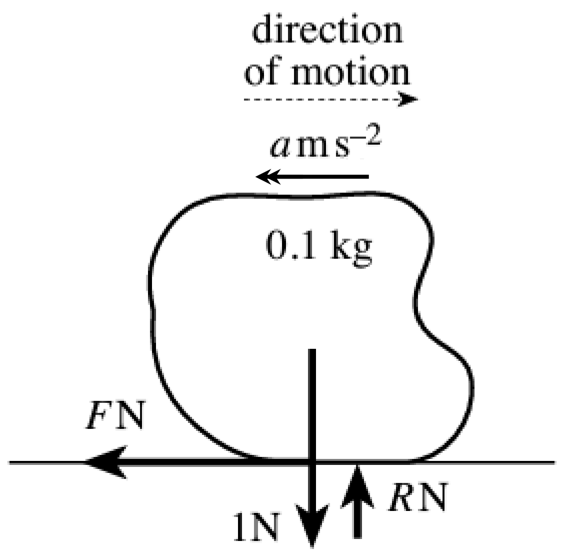
Example 5.2.2

A boy kicks a stone of mass 100 grams across the playground. The coefficient of friction between the stone and the playground is 0.25. If the stone comes to rest 31 m away, find the speed with which the boy kicked it.



Class exercises
Exercise 5A

13 A shopper picks up a 2 kg packet of rice with the thumb and index finger of one hand. The coefficient of friction between her fingers and the wrapping is 0.3. What horizontal force must she exert to prevent the packet from slipping?

2 The diagram shows horizontal forces of magnitudes P N and 100 N acting in opposite directions on a block of weight 50 N, which is at rest on a horizontal surface. Given that the coefficient of friction between the block and the surface is 0.4, find the range of possible values of P.

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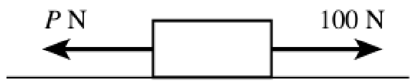
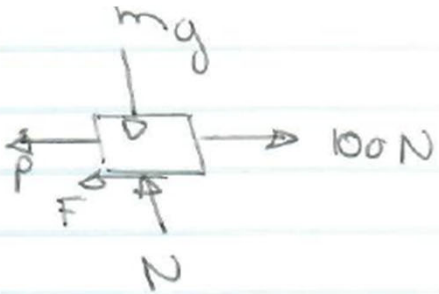


(2)



$+\uparrow)\ \Sigma F_y = 0$

$-50N + N = 0$

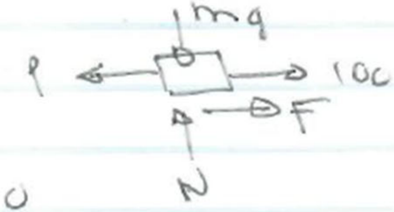
$\therefore N = 50\ N$

$\overset{+}{\rightarrow})\ -P - \mu N + 100 = 0$

$\therefore -P - 0.4(50) + 100 = 0$

$\therefore +P = 80\ N$ (For $P < 100$ N)

For $P > 100$:



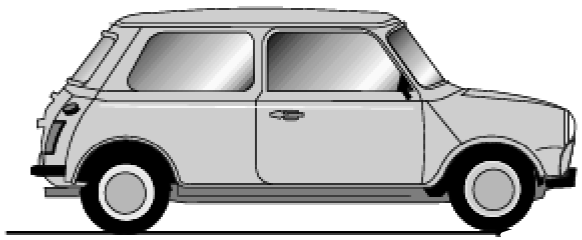
$\overset{+}{\rightarrow})\ -P + 0.4(50) + 100 = 0$

$\therefore P = 120\ N$

$\therefore 80 \le P \le 120\ N$

5.4 Friction and motion

Its direction is opposite to the direction of motion or possible motion.

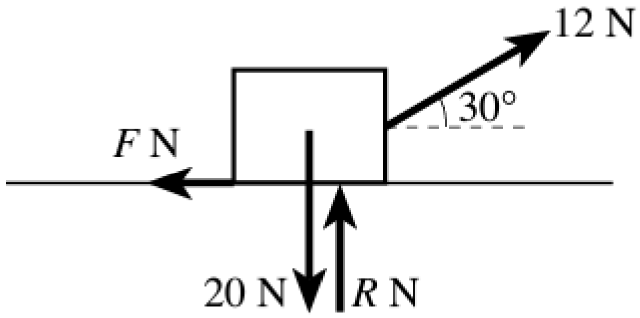


5.5 Problems involving friction

- If motion is taking place (whether at constant speed or with acceleration), friction is limiting and in a direction opposite to the direction of motion.
- If the object is on the point of moving, friction is limiting and in a direction opposite to that in which the object is about to move.
- If friction is limiting, then $F = \mu R$.
- Whether friction is limiting or not, $F \leqslant \mu R$.

Example 5.5.1

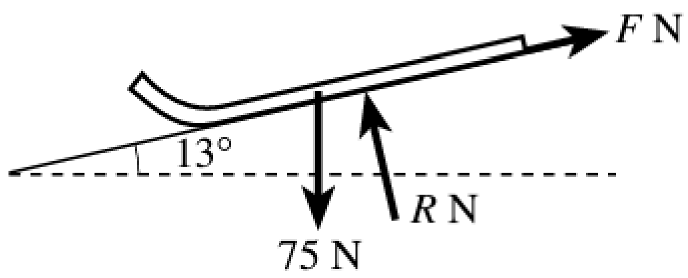
A block of weight 20 N is at rest on a horizontal surface. When a force of magnitude 12 N is applied to the block at an angle of $30°$ above the horizontal, it is on the point of moving. Find the coefficient of friction between the block and the surface.



A common mistake in situations like this is to think that $R = 20$ *. When resolving vertically, you must include every force with a vertical resolved part.*

Example 5.5.2

A snow-covered hill is at an angle of $13°$ to the horizontal. A sledge of weight 75 N is placed on the hill. Given that the coefficient of friction between the sledge and the hill is 0.15, find whether the sledge will slide down the hill by itself.



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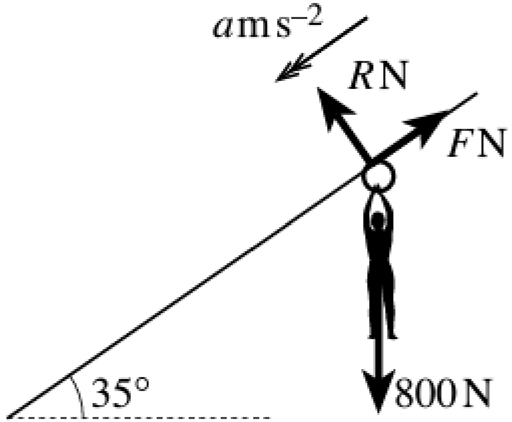
Example 5.5.3

Part of an army assault course consists of a taut cable 25 metres long fixed at 35° to the horizontal. A light rope ring is placed round the cable at its upper end. A soldier of mass 80 kg grabs hold of the ring and slides down the cable. If the coefficient of friction between the ring and the cable is 0.4, find how fast the soldier is moving when he reaches the bottom.



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Example 5.5.4

A block of weight 200 N is placed on a slope at $\beta°$ to the horizontal, where $\sin\beta° = 0.6$ and $\cos\beta° = 0.8$. It is kept from moving by a horizontal force of P newtons. For different values of the coefficient of friction μ, find the range of possible values of P.

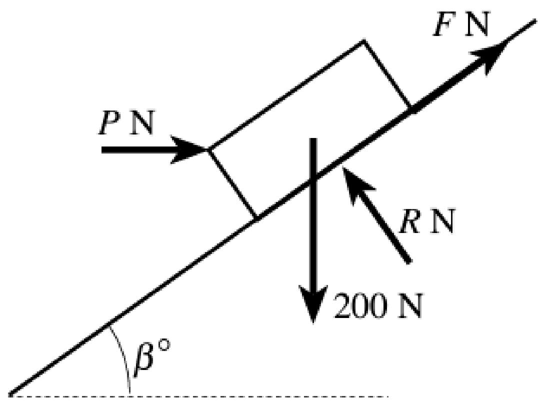


Fig. 5.17

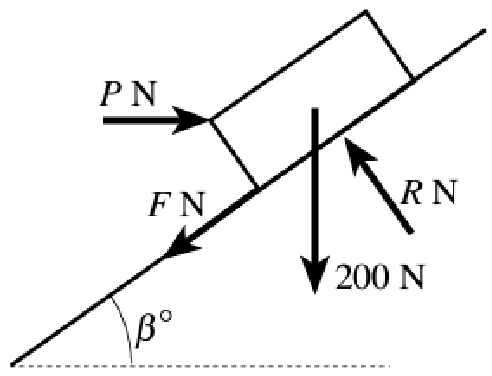
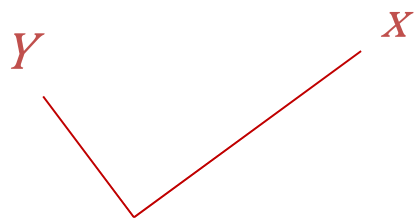


Fig. 5.18



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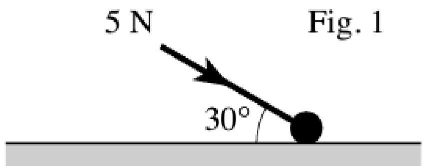
Class exercises
Exercise 5B

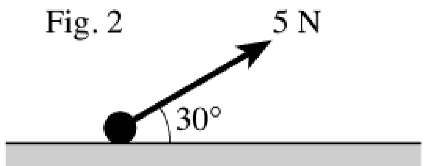
2 A cyclist and his bicycle have a total mass of 90 kg. He is travelling along a straight horizontal road, at 7 m s^{-1}, when he applies the brakes, locking both wheels. He comes to rest in a distance of 5 m. Find the coefficient of friction between the tyres and the road surface.

1 A book, which may be modelled as a particle of weight 8 N, rests in equilibrium on a desk top inclined at $28°$ to the horizontal. Find the frictional force acting on the book.

The coefficient of friction between the book and the desk top is 0.6. Determine whether the equilibrium is limiting. (OCR)

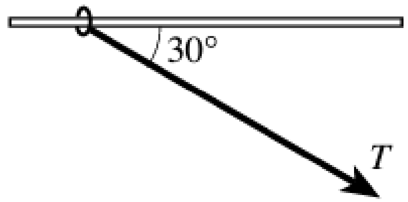
9





A small toy has mass 1.2 kg. When a child tries to move the toy along a horizontal floor by pushing with a force of magnitude 5 N, acting downwards at an angle of $30°$ to the horizontal as shown in Fig. 1, she is unsuccessful. When the child tries to move the toy by pulling with a force of magnitude 5 N, acting upwards at an angle of $30°$ to the horizontal as shown in Fig. 2, she is successful. Show that the coefficient of friction between the toy and the floor lies between 0.30 and 0.46, to 2 significant figures. (OCR, adapted)

3 A heavy ring of mass 5 kg is threaded on a fixed rough horizontal rod. The coefficient of friction between the rod and the ring is $\frac{1}{2}$. A light string is attached to the ring and pulled downwards with a force acting at a constant angle of $30°$ to the horizontal (see diagram). The magnitude of the force is T newtons, and is gradually increased from zero. Find the value of T that is just sufficient to make the equilibrium limiting. (OCR)



15 A wooden box is pulled along a rough horizontal floor by means of a constant force of magnitude 150 N acting at an angle of $40°$ above the horizontal. The box may be modelled as a particle of mass 45 kg, and air resistance may be neglected. Draw a diagram showing all the forces acting on the box, and show that the normal contact force of the floor on the box is approximately 354 N.

The coefficient of friction between the box and the floor is 0.3. Calculate the time taken for the box to move 40 m from rest. (OCR, adapted)