Homework assignments

Self study	**Homework (to be handed in)**
Work through Examples 7.6.1, 7.6.2 (Chapter 7) NB: Read Chapters 8 and 9	Exercise 8A: 5 Exercise 8B: 1, 3

8. Work, energy and power

- Know the definitions of work and kinetic energy
- Be able to use the work–energy principle, and distinguish work done by a force from work done against a force
- Know that the work–energy principle can be extended to situations in which forces are not constant
- Use the fact that a force perpendicular to the direction of motion does no work
- Understand the idea of power and be able to calculate it
- Know and be able to use the relation between power, force and speed.

8.1 The work–energy equation

Example 8.1.1

A car and driver have a total mass of 1000 kg. The car gains speed from 7 m s^{-1} to 13 m s^{-1} with constant acceleration over a distance of 200 metres. Calculate the driving force.

$$v^2 = u^2 + 2as$$

$$\therefore 13^2 = 7^2 + 2a(200)$$

$$\therefore a = 0.3 \text{ m/s}^2$$

$$\longrightarrow) \sum F = ma:$$

$$F_D = 1000(0.3) = 300 \text{ N}$$

$$v^2 = u^2 + 2as$$

$$a = \frac{(v^2 - u^2)}{2s} \qquad --(1)$$

$$F = ma \qquad --(2)$$

Sub. (1) into (2): $F = m\dfrac{(v^2 - u^2)}{2s}$

$$\therefore \frac{1}{2}mu^2 + Fs = \frac{1}{2}mv^2$$

Work–energy principle If a constant force acts on an object over a certain distance, the work done by the force is equal to the gain in the kinetic energy of the object.

$$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Fs: Work done by forces

$\frac{1}{2}mv^2$: Kinetic energy

$$\frac{1}{2}mu^2 + Fs = \frac{1}{2}mv^2$$

Joule (J)

$$(E_k)_1 + W = (E_k)_2$$

8.1 The work–energy equation

Example 8.1.1

A car and driver have a total mass of 1000 kg. The car gains speed from 7 m s^{-1} to 13 m s^{-1} with constant acceleration over a distance of 200 metres. Calculate the driving force.

$$\frac{1}{2}mu^2 + Fs = \frac{1}{2}mv^2$$

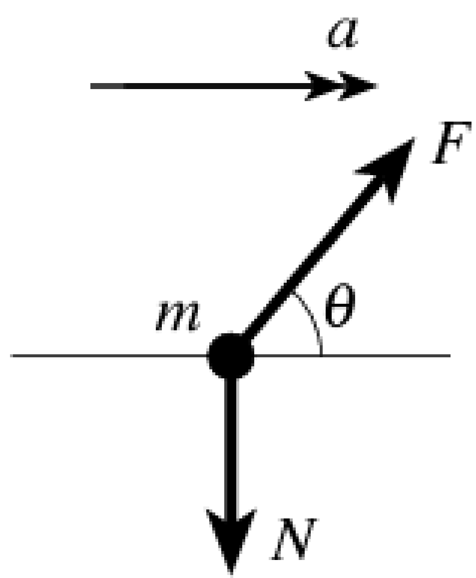
$$\therefore \frac{1}{2}(1000)7^2 + F_D(200) = \frac{1}{2}(1000)13^2$$

$$\therefore F_D = 300\text{ N}$$

8.2 Some generalisation

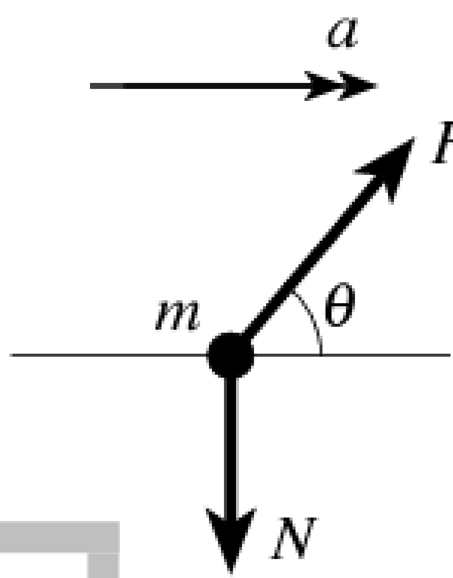
$$\frac{1}{2}mu^2 + Fs = \frac{1}{2}mv^2$$

$$\frac{1}{2}mu^2 + \qquad\qquad = \frac{1}{2}mv^2$$



8.2 Some generalisation

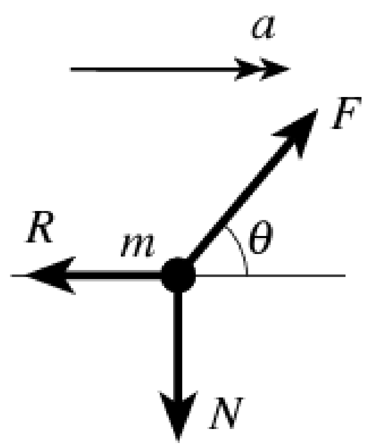
$$\frac{1}{2}mu^2 + Fs\cos\theta = \frac{1}{2}mv^2$$



If an object moves through a distance s along a line under the action of a force of magnitude F at an angle θ to the line, the work done by the force is $Fs\cos\theta$.

The work done by a force perpendicular to the direction of motion is zero.

$F s \cos\theta - R s =$ gain in kinetic energy



Extended work–energy principle The work done by the force acting on an object, minus the work done against resistance, is equal to the gain in the kinetic energy of the object.

$$F s \cos\theta - R s = \frac{1}{2} m v^2 - \frac{1}{2} m u^2$$

$$\frac{1}{2} m u^2 + F s \cos\theta - R s = \frac{1}{2} m v^2$$

Example 8.2.1

A cyclist and her machine together have a mass of 100 kg. She free-wheels down a hill of gradient 5% (1 in 20) for a distance of 500 metres. If her speed at the top was 5 m s^{-1}, and there is air resistance of 40 newtons, how fast will she be going at the bottom of the hill?

Example 8.2.2

A nail is being hammered into a plank. The mass of the hammer is 200 grams, and at each stroke the hammer is raised 15 cm above the nail. If the average force used to bring the hammer down is 10 times the average force used to raise the hammer, find the speed, to 2 significant figures, with which the hammer hits the nail.

8.3 Motion around curved paths

What work is done when a satellite orbits the earth?

Note that kinetic energy is a scalar!

$\frac{1}{2}mv^2$: Kinetic energy

Class exercises
Exercise 8A

6 An aircraft of mass 1.8 tonnes landing on an aircraft carrier at 144 kilometres per hour is brought to rest by a parachute brake and an arrester cable. If 30% of the work is done by the parachute, calculate the work done by the cable.

Miscellaneous exercises
Exercise 8

1 A small block is pulled along a rough horizontal surface at a constant speed of 2 m s^{-1} by a constant force. This force has magnitude 25 N and acts at an angle of $30°$ to the horizontal. Calculate the work done by the force in 10 seconds. (OCR)

8.4 Power

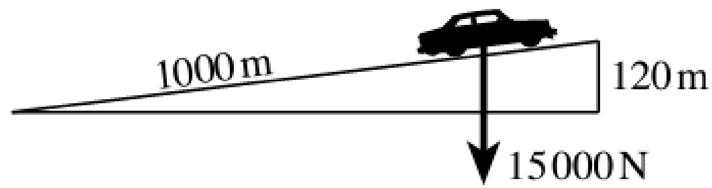
Competitors in a long-distance canoe race have to get past a series of waterfalls by dragging their canoes up the riverside path, a distance of 60 metres. One competitor takes 90 seconds to do this, another takes 100 seconds. The combined force of the friction and the resolved part of the weight down the path is 75 N for both canoes. Both competitors therefore do the same amount of work in raising their canoes, 75×60 J, which is 4500 J. But the first canoeist is more powerful; he does this work in a shorter time.

The rate at which work is done is called **power** $\therefore Power = \frac{work}{time}$

The unit for power is **watt (W)**

Example 8.4.1
A hotel lift, of total mass 1200 kg, rises a distance of 60 metres in 20 seconds. What is the power output of the motor?

Example 8.4.2
A car of mass 1500 kg arrives at the foot of a straight hill travelling at 30 m s^{-1}. It reaches the top of the hill 40 seconds later travelling at 10 m s^{-1}. The length of the hill is 1000 metres, and the gain in height is 120 metres. The average resistance to motion is 500 N. Find the average power developed by the engine.



$$Power = \frac{work}{time}$$

8.5 Power, force and velocity

Assume constant velocity v $\therefore s = vt$

$$W = Fs = Fvt$$

$$\therefore Power = \frac{W}{t} = \frac{Fvt}{t} = Fv$$

It can be proven that still holds even if the velocity is not constant.

> If an engine drives an object at velocity v by means of a force F in the direction of motion, the power developed by the engine is Fv.

Example 8.5.1

A swimmer of mass 50 kg pushes off from the side of a pool with a speed of 0.8 m s^{-1}. She can develop power of 200 W, and the resistance of the water is 220 N.

(a) At what rate can she accelerate away from the side of the pool?

(b) Assuming the resistance remains the same, what is her greatest possible speed?

Example 8.5.2

A racing car of mass 1830 kg is being tested out at high speeds. Running at full power, it is found that the greatest speed the car can achieve is 80 m s^{-1}. With the same power output, at a speed of 64 m s^{-1}, the car accelerates at 0.5 m s^{-2}. Assuming that the resistance to motion is proportional to the square of the speed, find the acceleration of the car at full power when its speed is 75 m s^{-1}.

Class exercises
Exercise 8B

4 A chair-lift runs at constant speed. Each passenger starts from rest at the lower station, and is delivered to the upper station with the speed of the lift. The lift raises 30 passengers a minute, of average mass 75 kg. The top of the lift is 300 metres higher than the bottom, and the ride takes 3 minutes. What power do the motors produce?