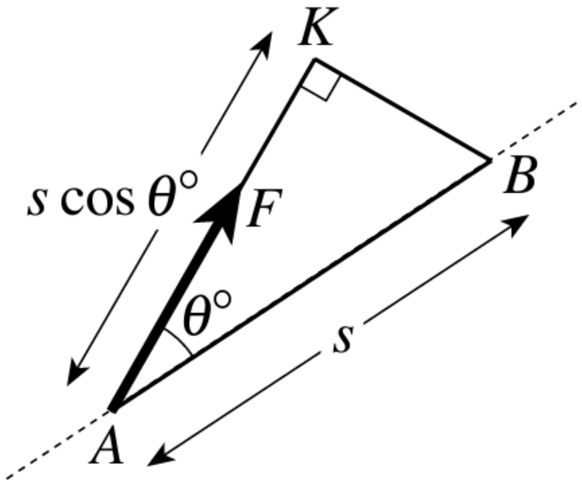
Homework assignments

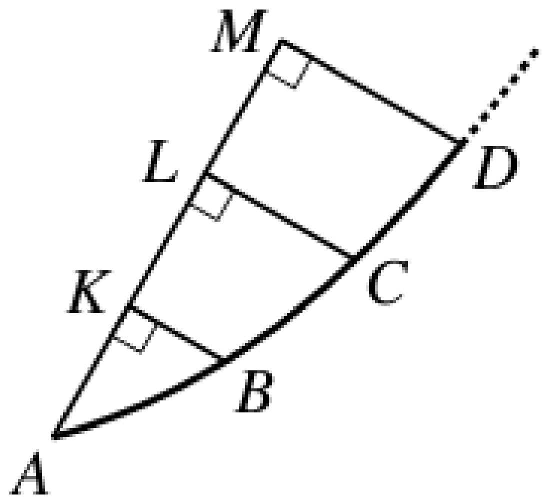
Self study	**Homework (to be handed in)**
NB: Read through chapters 8 and 9	Exercise 8B: 1, 3 Exercise 9A: 8, 13

9. Potential energy

- Know how to calculate the work done by a constant force acting on an object which moves in a curved path
- Know the difference between conservative and non-conservative forces
- Know the work done against a conservative force creates potential energy
- Understand and be able to apply the principle of conservation of energy
- Know that the total energy (potential and kinetic) can be changed by the work done by non-conservative forces

9.1 Another expression for work

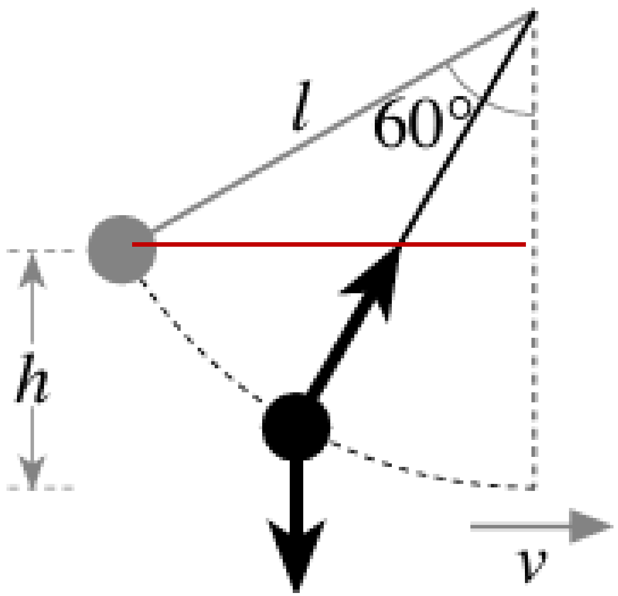




If an object moves along a curve under the action of a constant force of magnitude F, the work done by the force is equal to the product of F and the distance that the object moves in the direction of the force.

Example 9.1.1

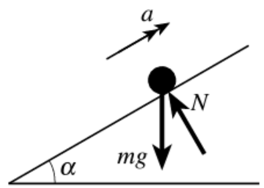
A small sphere of mass m is suspended from a hook by a thread of length l. The sphere is pulled sideways, so that the thread makes an angle of $60°$ with the downward vertical, and then released from rest. How fast is the sphere moving when the thread becomes vertical?



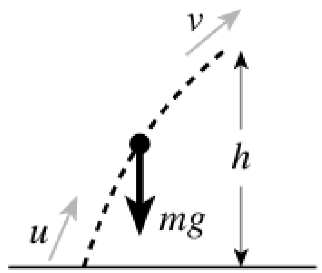
9.2 Three problems with one answer

1 A stone is thrown vertically upwards with initial speed u. Find its speed v when it has risen to a height h.

2 A block is hit and starts to move up a smooth path at an angle α to the horizontal. If its initial speed is u, find its speed v when it is at a height h above its starting point.



3 A ball is thrown at an angle to the horizontal with speed u. Neglecting the effect of air resistance, find how fast it will be moving when it is at a height h above the ground.



9.3 Conservative and non-conservative forces

4 A brick is set in motion with speed u across a rough floor. The frictional force is F. Find the speed at which it is moving when it has gone a distance h horizontally.

With friction the kinetic energy is lost due to friction.

When a ball is thrown upwards, kinetic energy is also "lost" (or transformed into potential energy rather) but can be gained again.

Conservative forces: Gravity, springs, etc.

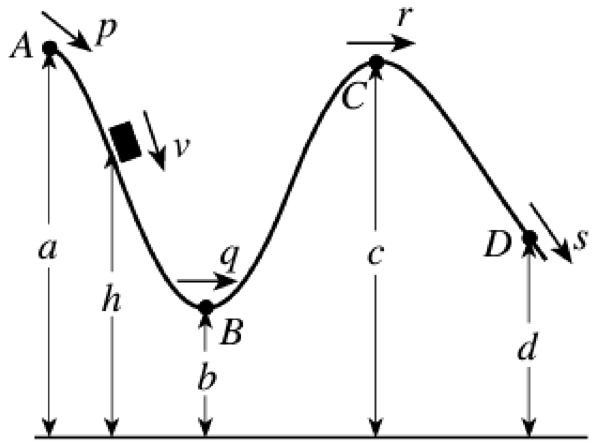
Non-Conservative forces: Friction, etc.

9.4 Conservation of energy

$$Wa + \tfrac{1}{2}mp^2 = Wb + \tfrac{1}{2}mq^2 .$$

$$Wb + \frac{1}{2}mq^2 = Wc + \frac{1}{2}mr^2 .$$

$$Wc + \frac{1}{2}mr^2 = Wd + \frac{1}{2}ms^2 .$$



$$(E_k)_1 + (E_p)_1 = (E_k)_2 + (E_p)_2$$

Conservation of energy principle For an object moving along a path, if there is no work done by external forces other than the force of gravity, the sum of the potential energy and the kinetic energy is constant.

Example 9.4.1

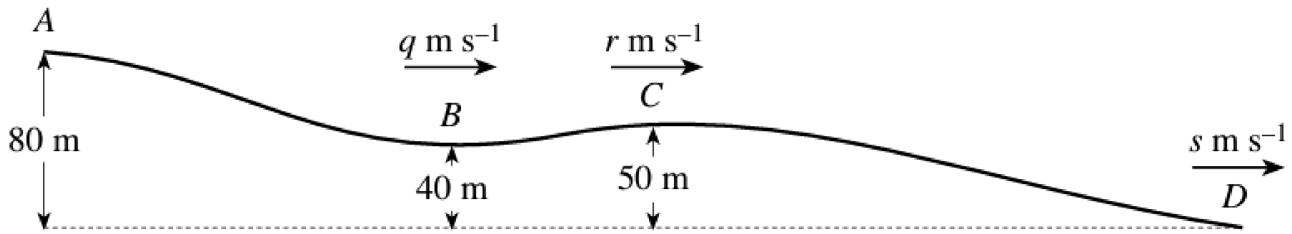


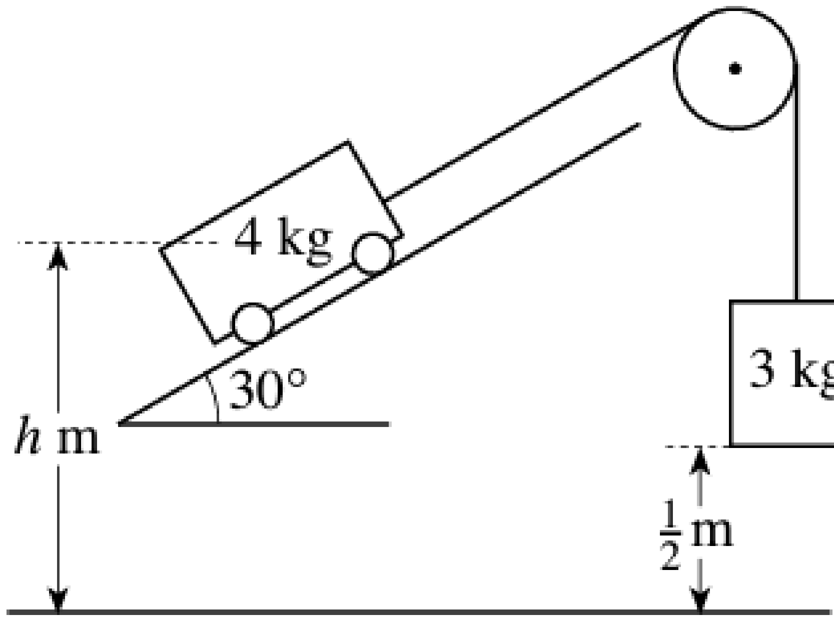
Fig. 9.8

Fig. 9.8 shows the profile of a hill. The points A, B and C are at heights 80, 40 and 50 metres respectively above D. A lightweight truck of mass 250 kg starts from rest at A and descends the hill without power. Neglecting any resistances, calculate how fast it is travelling at B, C and D.

9.4 Application to a system of connected objects

Example 9.5.1

A toy has the form of a truck of mass 4 kg which can run on a track at an angle of 30° to the horizontal. A light chain attached to the truck runs parallel to the track, passes over a light pulley at its upper end, and then hangs vertically (see Fig. 9.9). A counterweight of mass 3 kg is attached to the free end of the chain. The system is released from rest with the counterweight at a height $\frac{1}{2}$ m above the floor. Find how fast the truck is moving when the counterweight hits the floor.



9.5 Including non-conservative forces in the equations

Example 9.4.1

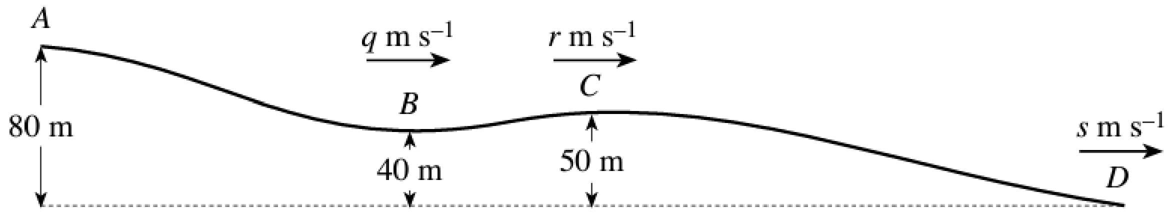


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Fig. 9.8 shows the profile of a hill. The points A, B and C are at heights 80, 40 and 50 metres respectively above D. A lightweight truck of mass 250 kg starts from rest at A and descends the hill without power. Neglecting any resistances, calculate how fast it is travelling at B, C and D.

Example 9.6.1

Suppose that in Example 9.4.1 the distances AB, BC and CD measured along the hill are 200 m, 100 m and 300 m. The average resistances to motion along these stretches of the hill are estimated to be 95 N, 140 N and 210 N. For the uphill stretch from B to C the motor is activated, producing a driving force of 700 N. Calculate the speeds at B, C and D using this model.

The rest for self study

For an object moving along a path, the total energy (potential and kinetic) is increased by the work done by the non-conservative external forces.

NOTE: Gravity is a conservative force and is already included as potential energy

"Positive" work will increase the energy whilst "negative" work will decrease the energy.

Example 9.6.2

Abe and Dev have mass 30 kg and 40 kg respectively. Abe is standing on the ground holding one end of a rope. Dev is 5 metres up a tree. Abe tosses the other end of his rope over a high branch. Dev grabs it, pulls it tight and then uses it to descend to the ground. As he does so, Abe keeps hold of the rope and goes up. As Dev reaches ground level, both boys are moving at 3 m s^{-1}. How much work is done against the frictional force between the rope and the branch?

Class exercises
Miscellaneous Ex. 9 pg. 141 (Old book)

2 A smooth plane AB is 10 metres long. It is inclined at $30°$ to the horizontal with the lower end, B, 6 metres vertically above horizontal ground. A particle is placed on the plane at the upper end, A, and then released from rest so that it slides down the plane. Find the speed of the particle as it strikes the ground. (OCR)

3 A skier of mass 70 kg sets off, with initial speed of 5 m s^{-1}, down the line of greatest slope of an artificial ski-slope. The ski-slope is 80 metres long and is inclined at a constant angle of $20°$ to the horizontal. During the motion the skier is to be modelled as a particle.

(a) Calculate the potential energy that the skier loses in sliding from the top to the bottom of the slope.

(b) Ignoring air resistance and friction, calculate the speed of the skier at the bottom of the slope.

The skier actually reaches the bottom of the slope with speed 6 m s^{-1}. Calculate the magnitude of the constant resistive force along the slope which could account for this final speed. (OCR)