Mechanics 1 Tutorial 2 9 October 2018

Question 1

A particle moves at constant acceleration in a straight line. It starts from rest and during the seventh second of its motion it covers 15 m.

(a) Determine the magnitude of the acceleration.

(b) Determine the speed of the particle after it has covered 100 m.

We are given that $v_0 = 0$ and that $x_2 - x_1 = 15\,\text{m}$, where x_2 is the distance travel from $x_0 = 0$ during the first 7 seconds, and similarly, x_1 is the distance travelled during the first 6 seconds.

a) Since we have constant acceleration, we may define x_1 and x_2 as

$$x_2 = 0 + 0(7) + \frac{1}{2}(a)(7)^2$$

$$x_1 = 0 + 0(6) + \frac{1}{2}(a)(6)^2$$

and consequently

$$x_2 - x_1 = \frac{1}{2}(a)(7)^2 - \frac{1}{2}a(6)^2$$

$$15 = a(\frac{1}{2}(7^2 - 6^2))$$

$$a = 2.308\,\text{ms}^{-2}$$

b) Here we must first determine how long it will take cover $x_3 = 100\text{m}$.

$$x_3 = x_0 + v_0 t \frac{1}{2}at^2$$

$$100 = 0 + 0(t) + \frac{1}{2}(2.308)t^2$$

$$t = 9.309\,\text{s}$$

Now we can find v_3 by the following formula:

$$x_3 - x_0 = \frac{1}{2}(v_0 + v_3)t$$

$$100 = \frac{1}{2}(v_3)(9.309)$$

$$v_3 = 21.48\,\text{ms}^{-1}$$

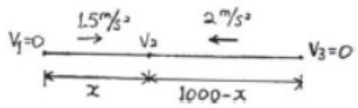
Question 2

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

Using formulas of constant acceleration:

$$v_2 = 1.5\, t_1$$

$$x = \frac{1}{2}(1.5)(t_1^2)$$



$$0 = v_2 - 2\, t_2$$

$$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$$

Combining equations:

$$t_1 = 1.33\, t_2; \quad v_2 = 2\, t_2$$

$$x = 1.33\, t_2^2$$

$$1000 - 1.33\, t_2^2 = 2\, t_2^2 - t_2^2$$

$$t_2 = 20.702 \text{ s}; \qquad t_1 = 27.603 \text{ s}$$

$$t = t_1 + t_2 = 48.3 \text{ s}$$

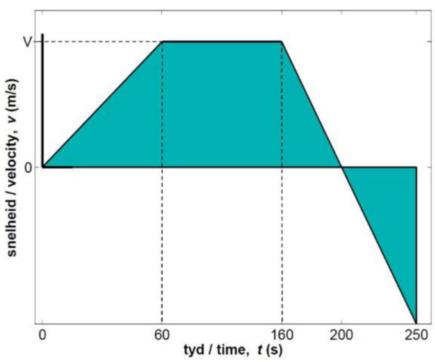
Question 3

The $v - t$ graph describes the velocity of a particle moving in a straight line. Determine V given that:

(a) the average velocity is 6.5 $\mathrm{m/s}$ in the positive direction.

(b) the average speed is 13 $\mathrm{m/s}$.

(c) the average acceleration between 60 and 250 seconds is $-0.5\ \mathrm{m/s^2}$.



Solution:

First determine the function of the line over the time interval $160 \le t \le 250$:

$$v(t) = -\frac{V}{40}(t - 200) = -\frac{V}{40}t + 5V$$

Therefore:

$$v(250) = -1.25V$$

(a)

$$\overset{+}{\rightarrow} \quad v_{avg} = \frac{\Delta s}{\Delta t}$$

$$6.5 = \frac{30(V) + 100(V) + 20(V) + 25(-1.25V)}{250}$$

$$V = 13.68 \text{ m/s}$$

(b)

$$(v_{sp})_{avg} = \frac{\Delta s}{\Delta t}$$

$$13 = \frac{30(V) + 100(V) + 20(V) + 25(1.25V)}{250}$$

$$V = 17.93 \text{ m/s}$$

(c)

$$\overset{+}{\rightarrow} \quad a_{avg} = \frac{\Delta v}{\Delta t}$$

$$-0.5 = \frac{-1.25V - V}{190}$$

$$V = 42.22 \text{ m/s}$$

Question 4

A lift moves upwards from rest and accelerates at 0.9 m s^{-2} for 3 s. The lift then travels for 6 s at constant speed and finally slows down, with a constant deceleration, stopping in a further 4 s.

(i) Sketch a velocity-time graph for the motion. [3]

(ii) Find the total distance travelled by the lift. [2]

(i)	Trapezium seen	**B1**	
	0, 3, 9, 13 shown on the *t* axis	**B1**	
	$v = 2.7$ soi in either part	**B1**	[3]
(ii)	[0.5 × (6 + 13) × 2.7]	**M1**	
	Total distance = 25.65 m	**A1**	[2]

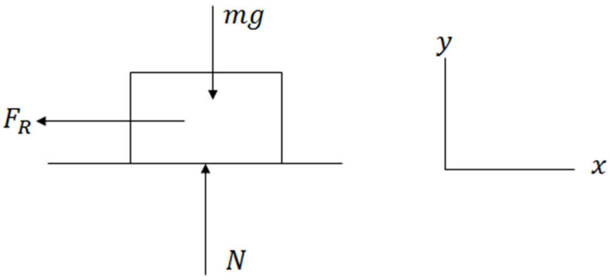
Question 5

A three ton truck (mass = 3000 kg) moves at 36 km/h on a straight road. It is brought to rest by a constant deceleration over a distance of 50 m.

(a) Determine the magnitude of the resistance force.

(b) Determine how long it takes the truck to come to rest.

(a) Solution:



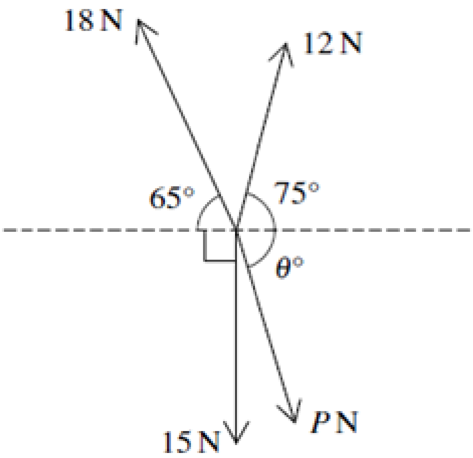
36 km/h = 36(1000/3600) = 10 m/s
Acceleration is constant:
$v^2 = v_0^2 + 2a(s - s_0)$
$\therefore 0 = 10^2 + 2a(50 - 0)$
$\therefore 0 = 10^2 + 2a(50 - 0)$
$\therefore a = -1\ \text{m/s}^2$
$\rightarrow)\ \sum F = ma\text{:}\quad F_R = 3000(-1) = -3000\ \text{N}$

(b) Solution:

$v = v_0 + at$
$\therefore 0 = 10 + (-1)t \qquad \Rightarrow t = 10\ \text{s}$

Question 6

The coplanar forces shown in the diagram are in equilibrium. Find the values of *P* and *θ*.



$12\cos 75^\circ + P\cos\theta^\circ = 18\cos 65^\circ$

$18\sin 65^\circ + 12\sin 75^\circ = 15 + P\sin\theta^\circ$

$[P^2 = (18\sin 65^\circ + 12\sin 75^\circ - 15)^2 + (18\cos 65^\circ - 12\cos 75^\circ)^2]$

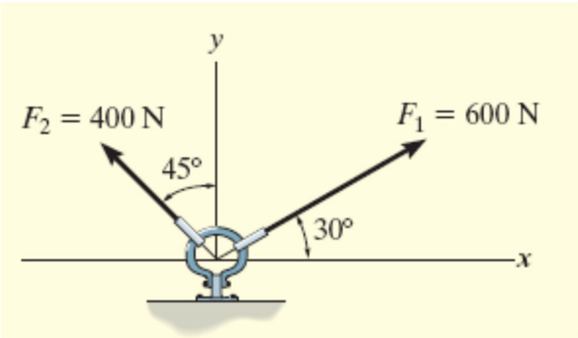
or

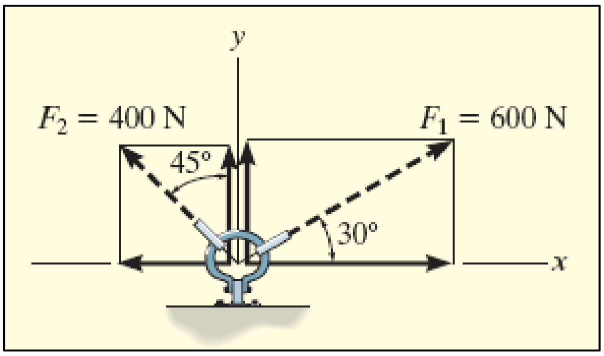
$[\theta = \tan^{-1}(18\sin 65^\circ + 12\sin 75^\circ - 15)/(18\cos 65^\circ - 12\cos 75^\circ)]$

$P = 13.7$ or $\theta = 70.8$

Question 7

The link is subjected to two forces **F**1 and **F**2. Determine the magnitude and orientation of the resultant force.





$F_{Rx} = \Sigma F_x :$

$F_{Rx} = 600\cos 30^\circ - 400\sin 45^\circ$

$= 236.8\,\text{N} \rightarrow$

$F_{Ry} = \Sigma F_y :$

$F_{Ry} = 600\sin 30^\circ + 400\cos 45^\circ$

$= 582.8\,\text{N} \uparrow$

Resultant Force

$F_R = \sqrt{(236.8)^2 + (582.8)^2}$

$= 629\,\text{N}$

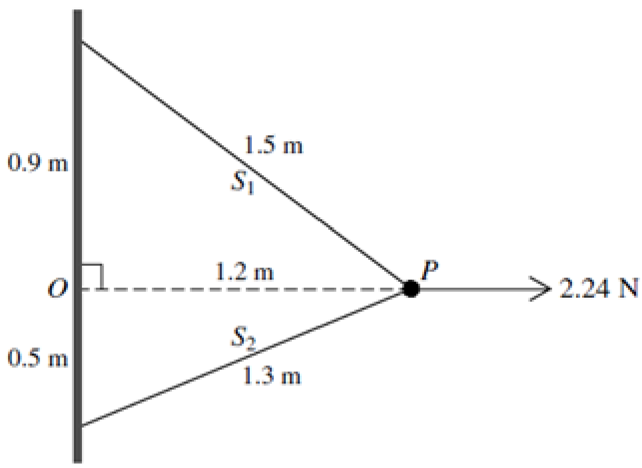
Angle θ is:

$\theta = \tan^{-1}\left(\frac{582.8}{236.8}\right)$

$= 67.9^\circ$

Question 8

A particle P of weight 1.4 N is attached to one end of a light inextensible string S_1 of length 1.5 m, and to one end of another light inextensible string S_2 of length 1.3 m. The other end of S_1 is attached to a wall at the point 0.9 m vertically above a point O of the wall. The other end of S_2 is attached to the wall at the point 0.5 m vertically below O. The particle is held in equilibrium, at the same horizontal level as O, by a horizontal force of magnitude 2.24 N acting away from the wall and perpendicular to it (see diagram). Find the tensions in the strings. [6]



$0.8T_1 + 12T_2/13 = 2.24$

$0.6T_1 - 5T_2/13 = 1.4$

$T_1 = 2.5$ and $T_2 = 0.26$