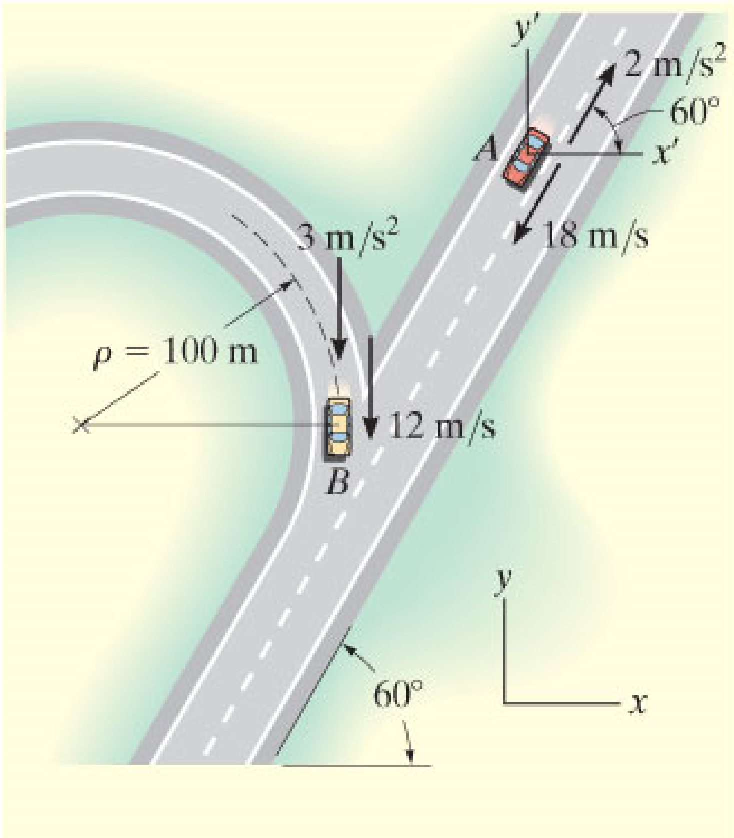
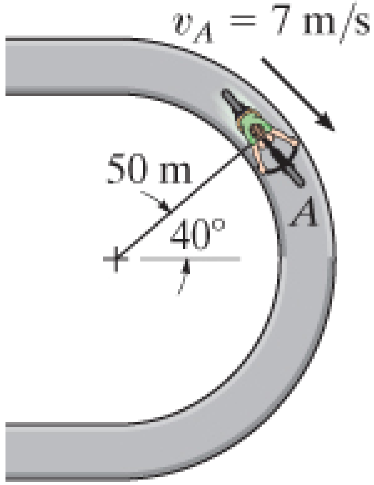
| Self study | Homework (to be handed in) |
|---|---|
| Read pp. 99 – 111<br><br>Work through all the examples in the chapter | Exercise 6A: 10<br>Exercise 6B: 1<br>Exercise 6C: 9 |

# 6. Motion round a circle

- Know the meaning of angular speed
- Know the equation connecting tangential speed and angular speed
- Know how to calculate the acceleration in circular motion with constant speed
- Be able to solve problems in two and three dimensions involving objects moving round a circular path.

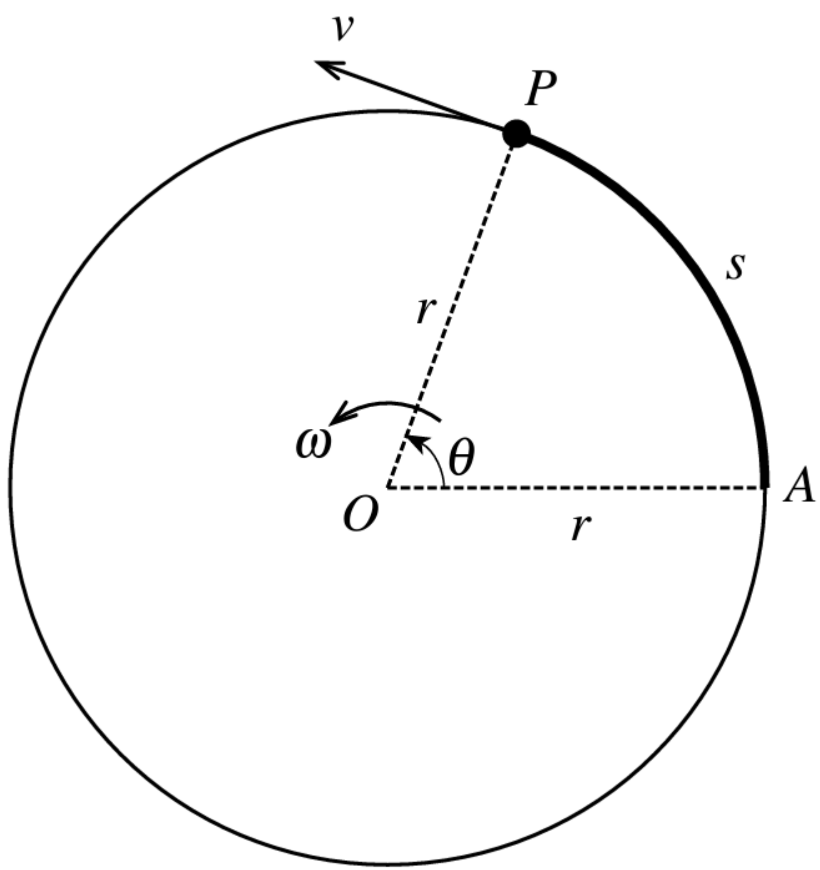
# 6.1 Example



# 6.2 Angular speed

Constant speed $v$




A particle moving round a circle of radius $r$ with angular speed $\omega$ has tangential speed $v$ given by $v = r\omega$.

**Example 6.2.1**
A car's tachometer records the engine speed as 3000 revolutions per minute. What is this in $\text{rad s}^{-1}$?

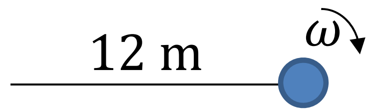
**Example 6.2.2**
Taking the orbit of the earth round the sun to be a circle of radius $1.495\times10^{11}$ metres, calculate the speed at which the earth is moving.

**Example 6.2.3**
The pilot of an aircraft flying at 800 km per hour on a bearing of $250°$ receives orders to change course to $210°$. The manoeuvre is completed in 20 seconds. Calculate the radius of the turn.

# Class exercises
# Exercise 6A pg. 103

**11** A flight simulator is modelled as a light rigid rod $OA$, of length 12 metres, with a particle of mass 75 kg attached to the end $A$. The rod rotates in a horizontal plane, about a fixed axis through $O$, with angular speed $\omega$ rad s$^{-1}$. Given that the kinetic energy of the particle is 14 000 J, find $\omega$. (OCR)

$$E_k = \frac{1}{2}mv^2 = 14000 \text{ J}$$

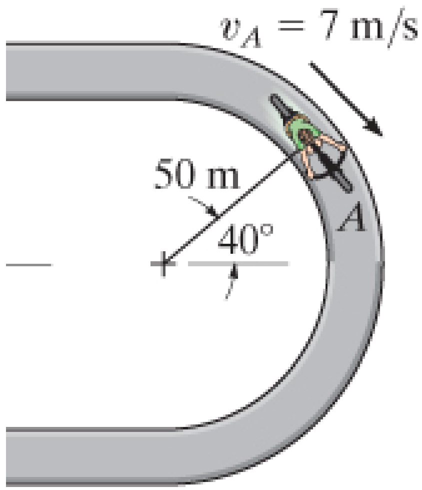



$$\therefore v = \sqrt{\frac{14000}{75}(2)} = 19.32 \text{ m/s}$$

$$\therefore v = \omega r$$

$$\therefore \omega = \frac{v}{r} = \frac{19.32}{12} = 1.61 \text{ rad/s}$$

# 6.3 Calculating the acceleration




A particle moving round a circular path of radius $r$ with constant angular speed $\omega$ and tangential speed $v$ has acceleration of magnitude $r\omega^2$, or $\frac{v^2}{r}$, directed towards the centre of the circle.

**Example 6.3.1**

Astronauts are trained to withstand the effects of high acceleration in a centrifugal machine. They sit or lie in cabins at the end of long metal arms, which rotate them about a vertical axis in a horizontal circle. If the radius of the circle is 12 metres, and the acceleration to be experienced is $10g$, how long should it take for the arms to make one revolution?

**Example 6.3.2**

Compare the acceleration of the moon towards the centre of the earth with the acceleration due to gravity at the earth's surface.

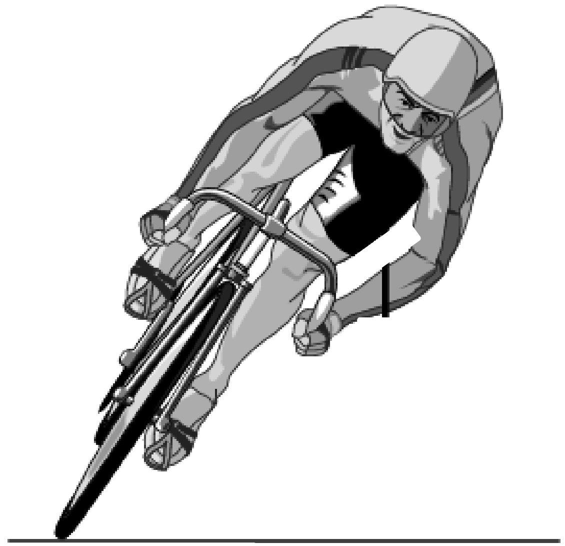
**Self study!**

**Example 6.3.3**

A smooth circular table of radius 1.2 metres has a raised rim. A ball-bearing of mass 50 grams runs round the rim of the table, making one circuit every 4 seconds. Find the magnitude of the contact force on the ball-bearing from the rim.

**Example 6.3.4**

A racing cyclist travels round a circle of radius 30 metres at a speed of $15 \text{ m s}^{-1}$. What must be the coefficient of friction between the tyres and the track for this to be possible?



**Example 6.3.4**

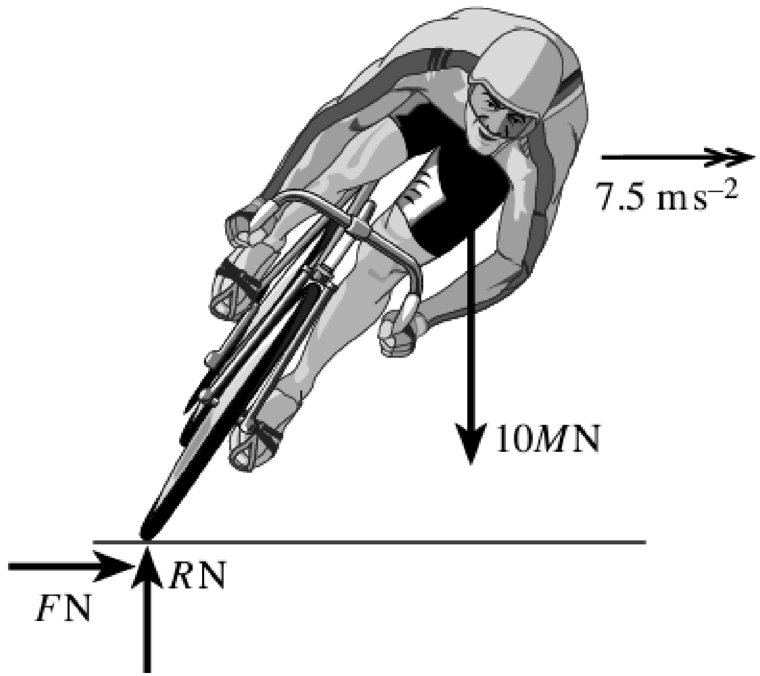
A racing cyclist travels round a circle of radius 30 metres at a speed of $15\ \text{m s}^{-1}$. What must be the coefficient of friction between the tyres and the track for this to be possible?

$$\rightarrow)\ \sum F_n = ma_n$$

$$\therefore F = M\frac{15^2}{30} = 7.5M$$

$$F_{\text{lim}} = \mu R \geq F = 7.5M$$

$$\therefore \mu \geq \frac{7.5M}{R} = \frac{7.5M}{10M} = 0.75$$



# Class exercises: Exercise 6B pg. 107

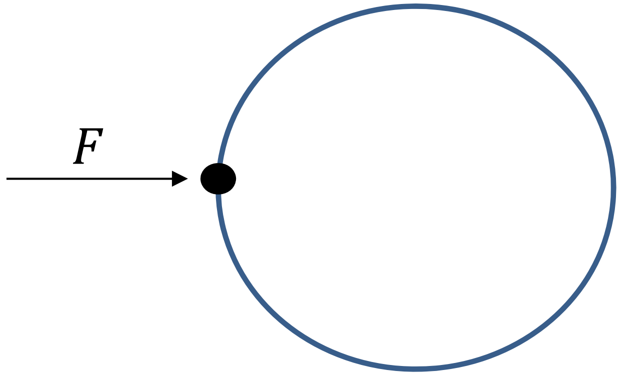
**9** A railway engine travels at a constant speed of $v$ m s$^{-1}$ on a curved track. The curve is an arc of a horizontal circle of radius 550 metres. The magnitude of the acceleration of the engine is 0.22 m s$^{-2}$. Making a suitable modelling assumption, which should be stated, calculate $v$.

The mass of the engine is 45 000 kg. Calculate the magnitude of the resultant horizontal force on the engine. (OCR)

$$a_n = \frac{v^2}{r}$$

$$\therefore v^2 = r\,a_n = 550(0.22)$$

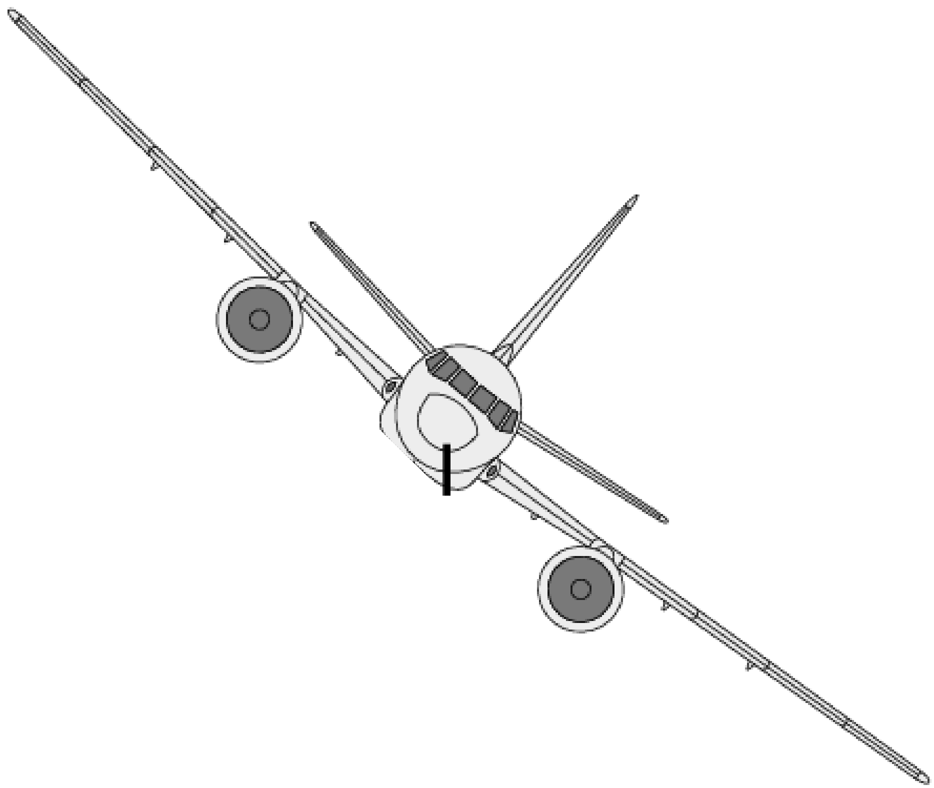
$$\therefore v = 11 \text{ m/s}$$




$$\rightarrow) \sum F_n = m a_n$$

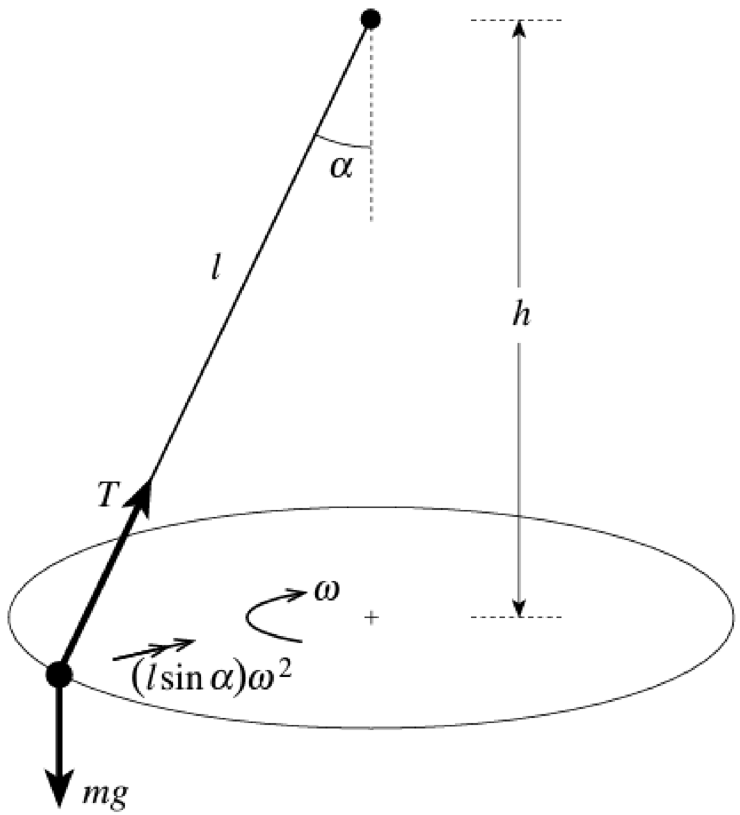
$$\rightarrow F = 45000(0.22) = 9900 \text{ N}$$

# 6.5 Three-dimensional problems



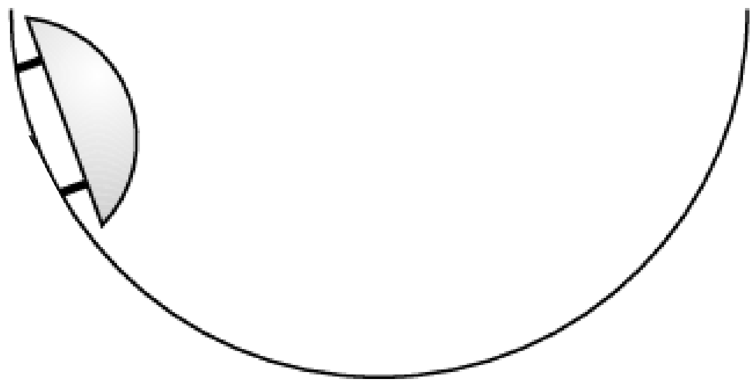
**Example 6.5.1**

One end of a string of length $l$ is tied to a hook, and a particle of mass $m$ is attached to the other end. With the string taut and making an angle $\alpha$ with the downward vertical, the particle is set in motion so that it rotates in a horizontal circle about the vertical line through the hook. Find the period of one revolution of the particle round the circle.



**Example 6.5.2**

A bob-sleigh with its two-person team has a total mass of 200 kg. On one stretch of the course the team rounds a horizontal bend of radius 25 metres at a speed of $35\ \mathrm{m\ s^{-1}}$. They bank the sleigh so that it rounds the bend with no sideways frictional force. Calculate the acceleration of the sleigh, and find the angle to the horizontal at which the sleigh is banked.



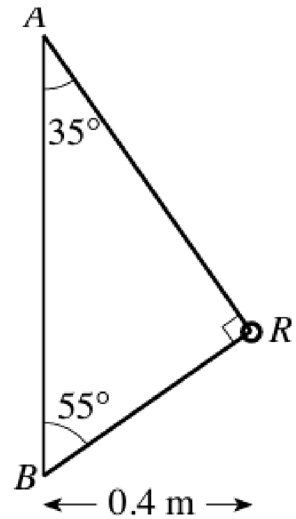
**Self study!**

# Class exercises
# Exercise 6C pg. 111

**7** A small smooth ring $R$, of mass $0.4$ kg, is threaded on a light inextensible string. The ends are attached to two fixed points $A$ and $B$, where $A$ is vertically above $B$. The system rotates about $AB$. The ring $R$ moves with constant speed in a horizontal circle of radius 0.4 metres. Angle $ARB$ is $90°$ and angle $BAR$ is $35°$ (see diagram).

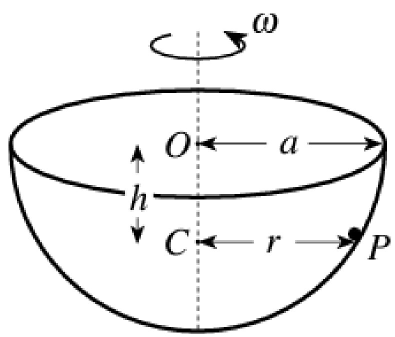
(a) Find the tension in the string.

(b) Find the speed of the ring. (OCR)



# Exercise 6C: 4, pg. 111

**4** A particle $P$ of mass $m$ kg moves on the smooth inner surface of a fixed hollow hemisphere with centre $O$, radius $a$ metres and axis vertical. The particle moves in a horizontal circle with centre $C$ and radius $r$ metres, and $CP$ rotates with angular speed $\omega$ rad s$^{-1}$. The distance $OC$ is $h$ metres (see diagram). Show that $\omega^2 = \dfrac{g}{h}$, and express the magnitude of the normal contact force between $P$ and the surface in terms of $m$, $g$, $a$ and $h$.




(OCR)