Question 1

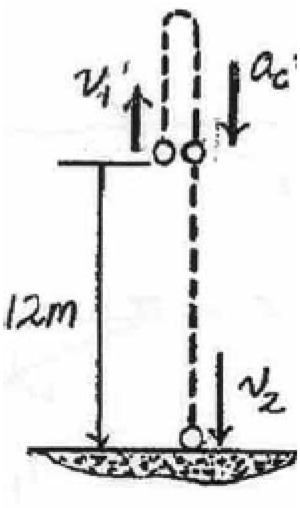
A boy throws a ball straight up from the top of a 12-m high tower. If the ball falls past him 0.75 s later, determine the velocity at which it was thrown, the velocity of the ball when it strikes the ground, and the time of flight.

When the ball passes the boy:

$\uparrow)\ y = v_0\Delta t + \frac{1}{2}a\Delta t^2$

$\therefore 0 = v_0(0.75) + \frac{1}{2}(-10)(0.75)^2$

$\therefore v_0 = 3.75 \text{ m/s}$



When the ball strikes the ground:

$\uparrow) y = v_0\Delta t + \frac{1}{2}a\Delta t^2$

$\therefore -12 = 3.75(\Delta t_2) + \frac{1}{2}(-10)(\Delta t_2)^2$

$\therefore 5(\Delta t_2)^2 - 3.75(\Delta t_2) - 12 = 0$

$\therefore \Delta t_2 = 1.97 \text{ s}$

$\uparrow)\ v = v_0 + at$

$\therefore v_2 = 3.75 + (-10)(1.97) = -15.94 \text{ m/s} = 15.94 \text{ m/s } \downarrow$

Question 2

A small ball B is projected with speed 15 ms^{-1} at an angle of 41° above the horizontal from a point *O* which is 1.6 m above horizontal ground. At time *t* s after projection the horizontal and vertically upward displacements of *B* from *O* are *x* m and *y* m respectively.

(i) Express *x* and *y* in terms of *t* and hence show that the equation of the trajectory of *B* is

$$0.869x - 0.0390x^2,$$

where the coefficients are correct to 3 significant figures.

A vertical fence is 1.5 m from *O* and perpendicular to the plane in which *B* moves. *B* just passes over the fence and subsequently strikes the ground at the point *A*.

(ii) Calculate the height of the fence, and the distance from the fence to A.

(i)

$\rightarrow) x = v_0 \cos\theta\, t = 15\cos 41°\, t \qquad (or \quad x = 11.321t)$

$\therefore t = \dfrac{x}{15\cos 41°} \qquad ------(1)$

$y = v_0 \sin\theta\, t + \dfrac{1}{2}at^2 = 15\sin 41°\, t + \dfrac{1}{2}(-10)t^2 \qquad ----(2)$

Sub (1) into (2): $y = 15\sin 41° \left(\dfrac{x}{15\cos 41°}\right) + \dfrac{1}{2}(-10)\left(\dfrac{x}{15\cos 41°}\right)^2$

$y = 0.869x - 0.0390x^2$

(ii)

Height of fence/ hoogte van heining:

$x = 1.5$ m: $h = 0.869(1.5) - 0.0383(1.5)^2 + 1.6 = 2.82$ m

Distance from the fence to *A*/ afstand vanf heinging tot *A*:

$y = -1.6$: $-1.6 = 0.869x - 0.0390x^2$

$\therefore x = 23.99$

Distance/ afstand: $D = 23.99 - 1.5 = 22.5$ m

Question 3

A particle *P* is projected with speed 15 ms^{-1} at an angle of 60° above the horizontal. Find the direction of motion of *P* at the instant 0.9 *s* after projection.

v_v = 15sin60 – 0.9 g	B1		3.99
	M1		Ratio of vert and horiz speeds
$\tan\theta$ = (15sin60-0.9g)/(15cos60)	A1		
θ =28(.0)° above horizontal	A1	[4]	

Question 4

A ball is projected horizontally with speed 5 ms^{-1} from the top of a tower which is 30 m high. The tower stands on horizontal ground.

(i) Find the speed and direction of motion of the ball when it reaches the ground.

(ii) Calculate the distance from the foot of the tower to the point where the ball reaches the ground

v = 25 ms^{-1}	B1		$\sqrt{(5^2 + 2\,g \times 30)}$
$\cos\theta$ = 5/25	M1		Forms a relevant trig ratio
θ = 78.5° (with horizontal)	A1	[3]	Ignore above/below
30 = $gt^2/2$	M1		t = 2.45, award if found in (i)
s = 5 × 2.45	M1		5 × time of flight
s = 12.2 m	A1	[3]	

Question 5

A small ball *B* is projected with speed U m/s at an angle of $\theta°$ above the horizontal from a point *O*. At time 2 s after the instant of projection, *B* strikes a smooth wall which slopes at 60° to the horizontal. The speed of *B* is 18 m/s and its direction of motion is perpendicular to the wall at the instant of impact (see Fig. 1). *B* bounces off the wall with speed V m/s in a direction perpendicular to the wall. At time 0.8 s after *B* bounces off the wall, *B* strikes the wall again at a lower point *A* (see Fig. 2).

(i) Find U and θ.

(ii) By considering the motion of *B* after it bounces off the wall, calculate *V*.

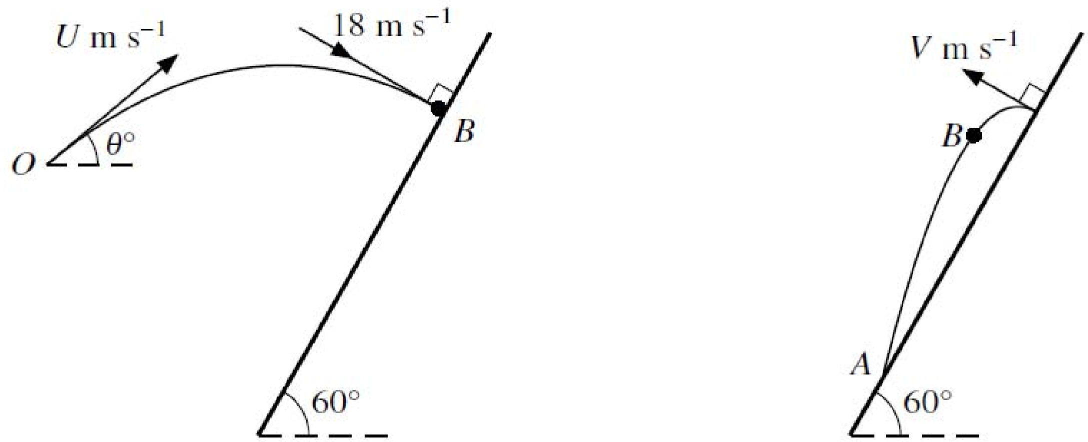


Fig. 1 **Fig. 2**

(i)

Speed in horizontal direction is constant:

$\rightarrow)\ v_x = U\cos\theta : U\cos\theta = 18\cos 30°\ \left(= 9\sqrt{3} = 215.588\ ...\right)$ ----- (1)

$\uparrow) v_y = U\sin\theta - (10)t:\ -18\sin 30° = U\sin\theta - (10)(2)$

$\therefore U\sin\theta = (10)(2) - 18\sin 30° = 11$ ------(2)

$(1)^2 + (2)^2: U^2(\cos^2\theta + \sin^2\theta) = 81(3) + (11)^2 = 364$

$\therefore U = 19.1$ m/s

$\therefore \theta = 35.2°$

(ii)

$\rightarrow)\ x = v_0 t = V\cos 30°\,(0.8)$ ----- (1)

$\uparrow)\ y = V\sin 30°\,(0.8) - \frac{1}{2}(10)(0.8)^2$ ----- (2)

When ball strikes wall/ wanneer bal die muur tref:

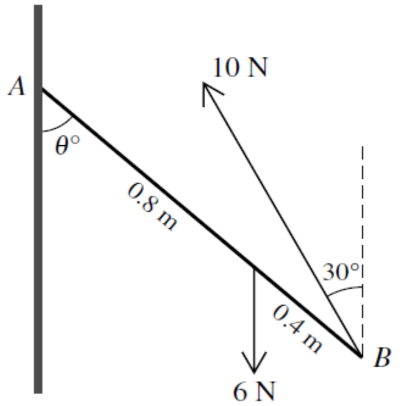
$$\frac{-y}{x} = \tan 60°$$

$$\therefore \frac{-V\sin 30°\,(0.8) + \frac{1}{2}(10)(0.8)^2}{V\cos 30°\,(0.8)} = \tan 60°$$

$$\therefore V = 2 \text{ m/s}$$

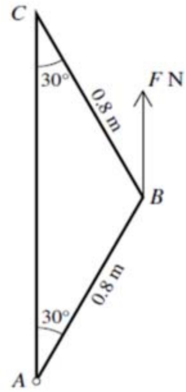
Question 6

A non-uniform rod AB of weight 6 N rests in limiting equilibrium with the end A in contact with a rough vertical wall. AB = 1.2 m, the centre of mass of the rod is 0.8 m from A, and the angle between AB and the downward vertical is $\theta°$. A force of magnitude 10 N acting at an angle of 30° to the upwards vertical is applied to the rod at B (see diagram). The rod and the line of action of the 10 N force lie in a vertical plane perpendicular to the wall. Calculate the value of θ



$10\cos30 \times 1.2\sin\theta - 10\sin30 \times 1.2\cos\theta$	M1		
$= 6 \times 0.8\sin\theta$	A1		
$5.5923..\sin\theta = 6\cos\theta$	M1		Creating a 3 term solvable equation in
$\theta = 47(.0)$	A1	4	$\sin\theta$ and $\cos\theta$
OR			
$10 \times 1.2\sin(\theta - 30) = 6 \times 0.8\sin\theta$ or	M1		
$10 \times 1.2\cos(120 - \theta) = 6 \times 0.8\sin\theta$	A1		
$5.5923..\sin\theta = 6\cos\theta$	M1		Creating a 3 term solvable equation in
$\theta = 47(.0)$	A1		$\sin\theta$ and $\cos\theta$

Question 7



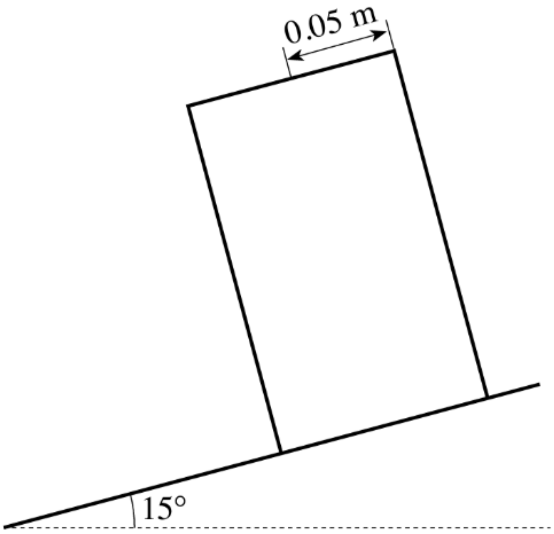
A triangular frame ABC consists of two uniform rigid rods each of length 0.8 m and weight 3 N, and a longer uniform rod of weight 4 N. The triangular frame has $AB = BC$, and angle BAC = angle $BCA = 30°$.
The vertex A of the frame is attached to a smooth hinge at a fixed point. The frame is held in equilibrium with AC vertical by a vertical force of magnitude F N applied to the frame at B.

(i) Calculate the distance of the centre of mass of the frame from AC.
(ii) Calculate F, and state the magnitude and direction of the force acting on the frame at the hinge.

(i)	$d(3+3+4) = 3 \times 0.4\sin 30 \times 2$	M1 A1		Taking moments about AC
	$d = 0.12$ m	A1	3	
(ii)	$(3+3+4) \times 0.12 = F \times 0.8\sin 30$	M1		Taking moments about A, allow candidate's d
	$F = 3$	A1		
	At hinge, 7 N upwards	B1√	3	Ft 10 – candidate's value (F) (downwards if negative)

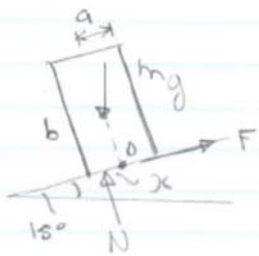
Question 8

A uniform cylinder of radius 0.05 m is held on a rough plane inclined at 15° to the horizontal (see diagram). The coefficient of friction between the plane and the end of the cylinder in contact with it is 0.3. The cylinder is released from rest. Determine whether or not the cylinder remains in equilibrium in each of the following cases:



(a) the height of the cylinder is 0.4 m;
(b) the height of the cylinder is 0.35 m.

(If in either case, the cylinder does not remain in equilibrium, you should state with a reason the way in which the cylinder starts to move.)



$a = 0{,}05$ m

(↗+) $\Sigma F = 0$

$mg\sin 15^\circ - F = 0$ — ①

(↖+) $\Sigma F = 0$: $N - mg\cos 15^\circ = 0$

$\therefore N = mg\cos 15^\circ$ — ②

(↺+) $\Sigma M_O = 0$: $mg\sin 15^\circ (b/2) - Nx$

$\therefore mg\sin 15 (b/2) - mg\cos 15^\circ x = 0$

$x = \frac{b}{2}\tan 15^\circ$

(a) $b = 0{,}4$ m: $x = 0{,}0536 \text{ m} > 0{,}05$ m

Topples

(b) $b = 0{,}35$ m: $x = 0{,}047 \text{ m} < 0{,}05$ m

From ①: $F = mg\sin 15^\circ = 2{,}54(m)$

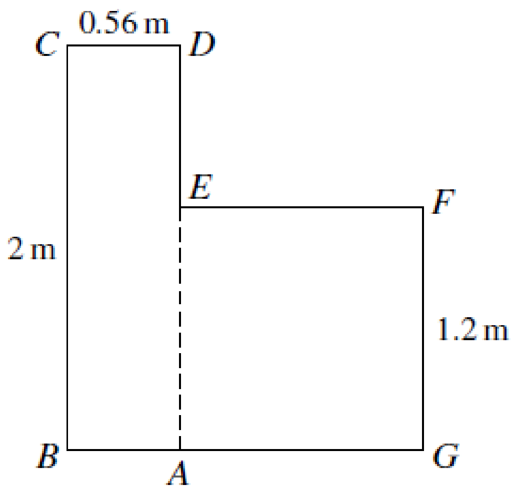
$F_{max} = \mu N = 0{,}3(mg\cos 15^\circ) = 2{,}9(m)$

$\therefore F < F_{max} \Rightarrow$ Does not slip

$\Rightarrow$ Equilibrium

Question 9

A uniform lamina is made by joining a rectangle *ABCD*, in which *AB* = *CD* = 0.56 m and *BC* = *AD* = 2 m, and a square *EFGA* of side 1.2 m. The vertex *E* of the square lies on the edge *AD* of the rectangle (see diagram). The centre of mass of the lamina is a distance *h* m from *BC* and a distance *v* m from *BAG*.



(i) Find the value of *h* and show that $v = h$.

The lamina is freely suspended at the point B and hangs in equilibrium.

(ii) State the angle which the edge *BC* makes with the horizontal.

Instead, the lamina is now freely suspended at the point *F* and hangs in equilibrium.

(iii) Calculate the angle between *FG* and the vertical.

(i)	$2 \times 0.56 \times 0.28 + 1.2^2(0.56 + 1.2/2) = h(2 \times 0.56 + 1.2^2)$	**M1**		Moments about BC
	$h = 0.775$	**A1**		
	$2 \times 0.56 \times 1 + 1.2^2(1.2/2) = v(2 \times 0.56 + 1.2^2)$	**M1**		Moments about BAG
	$v = 0.775$	**A1**	4	
(ii)	45°	**B1**	1	
(iii)	$\tan\theta = (0.56 + 1.2 - 0.775) / (1.2 - 0.775)$	**M1**		
	$\theta = 66.7°$	**A1**	2	